Field Theory and the Standard Model

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Abstract

The following 5 lectures are devoted to key ideas in field theory and in the Standard Model.

Lecture I. **Quantum Field Theory. Bird's-eye-view.**

Introductory remarks.

Quantum Field Theory (QFT) is the working language in the community of high energy physicists. It is not the esoteric theory accessible to the small group of experts, the basic ideas of QFT have to be familiar to all members of the community.There are number of excellent textbooks on QFT. The list of the recommended books can be found in ref [1]. As a rule they are rather lengthy, the average size of the standard textbook is of the order of 800 pages. The only way to understand QFT is to take one of these books and to spend one year or more to study it. It is hard way, but nobody knows the better one. So is life!

The goal of these lectures is not to provide any systematic introduction to the subject. It is impossible to do in five lectures. The goal is to remind the students what they actually had studied a few years ago. Just the basic concepts, notions and relations of QFT without long derivations and boring formalism.

1.1. Particles and Fields.

In the Classical Physics the particles and fields are very different dynamical systems. Particles are particles and fields are fields. There is no way to confuse these notions. This is everyday wisdom.

The system of particles has finite number of degrees of freedom N. To describe this dynamical system one have to introduce the general coordinates $q_i(t)$ $(i = 1, 2, ..N)$ and their time derivatives $\dot{q}_i(t)$ or conjugated momenta $p_i(t)$. Either we study the bounded motion or the scattering processes at any time we can say how many degrees of freedom the system has. Even when we observe the decay of the system we are sure that outgoing particles were bounded inside the initial system before the decay. This is evident.

Field theory is a theory of the system with infinite number of degrees of freedom. To describe the electromagnetic fields we have to know four-potential A_μ at every space point x. Maxwell equations govern the evolution of the field in time.

The Quantum Mechanics (QM) of <u>nonrelativistic</u> particles was developed in 1925-26. In QM dynamical system with N degrees of freedom is described

by wave function

$$\Psi(q,t) = \Psi(q_1,...,q_N;t) \tag{1.1}$$

that satisfies the wave equation

$$i\frac{\partial}{\partial t}\Psi(q,t) = H(p,q)\Psi(q,t) \tag{1.2}$$

where $H(p,q)$ is the Hamiltonian and p is the operator of momentum: $p = -i\partial/\partial q$. The number of degrees of freedom N was supposed to be fixed exactly like in Classical Mechanics.

The first quantization of electromagnetic fields had been done at the same time in 1926 by Born, Heisenberg and Jordan in their second paper on QM. They represented radiation electromagnetic field as an infinite set of harmonic oscillators and quantized these oscillators. They found that excitations of the oscillators behave like a free massless particles – photons. The number of photons was not fixed. They were created and destroyed by charged particles. Quantized theory of electromagnetic field became a theory of particles – photons. Photons were not "bounded" inside charged particles, they were created from "nothing" by scattered charged particles. The physical idea of photons was introduced by Einstein twenty years before this paper, but the formal quantization of field showed that quantized field is equivalent to the system of particles that can be created and destroyed.

For some time physicists tried to find a relativistic version of wave equation (1.2) for the particles at high energy.

The first such equation was written in 1926 by Klein and Gordon for spin 0 relativistic particle

$$-\partial_\mu\partial_\mu\Phi(x) = m^2\Phi(x) \tag{1.3}$$

where $\partial_\mu = \partial/\partial x_\mu$ and $\Phi(x)$ is a complex function of $x = (t,\vec{x})$, m is a mass of particle.

Dirac pointed out that eq. (1.3) and the function $\Phi(x)$ can't be interpreted as a wave equation and wave function. In 1928 he suggested his own relativistic equation for spin 1/2 particles:

$$(i\gamma_\mu\partial_\mu - m)\Psi(x) = 0 \tag{1.4}$$

Here Ψ is a column with 4 complex components (4-spinor) and γ_μ are 4×4 matrices.

The troubles with interpretation of eq. (1.4) as the one-particle relativistic wave equation are not so evident as for the case of eq. (1.3). But

the truth is that for any relativistic processes the single particle description breaks down. Any relativistic system has infinite numbers of degrees of freedom. More energy we pump into the system, more degrees of freedom can be excited. For example any scattering process in QED can be accompanied by creation of additional e^+e^- pairs. These pairs are not hidden inside initial particles, they are created during the scattering process. The natural description of relativistic physics is quantum field theory. So it is wrong to divide world on particles and fields. We have to use the quantum field theory for everything.

From this point of view both the Klein-Gordon and Dirac equations are not relativistic wave equations. They are field equations for scalar and spinor fields. These fields have to be quantize. The lowest excitations of these quantum fields behave like massive particles with spin 0 and 1/2 respectively.

The QFT is the right language for dealing with particle physics. This language is not unique. For example string theory also pretends to describe particles in low energy limit. We have also to note that one can construct "diagrammatica" (i.e. the set of rules for calculation of amplitudes in perturbation theory) without any reference to QFT.

1.2. Quantization and the Fock space.

Consider a field theory for a free scalar particle:

$$\Phi(x) = \Phi(\vec{x}, t) \tag{1.5}$$

In the "Classical Theory" $\Phi(x)$ is a real function of space-time point $x_\mu = (t, \vec{x})$ with the lagrangian density $\mathcal{L}(\Phi, \partial_\mu\Phi)$

$$\mathcal{L}(\Phi, \partial_\mu\Phi) = \frac{1}{2}\{\partial_\mu\Phi\partial_\mu\Phi - m^2\Phi^2\} \tag{1.6}$$

where $\partial_\mu\Phi = \frac{\partial}{\partial x_\mu}\Phi$ and m is the mass of the particle.

The action S is given by

$$S = \int d^4x\mathcal{L}(\Phi, \partial_\mu\Phi) \tag{1.7}$$

The hamiltonian density is constructed according to the rules of hamiltonian dynamics

$$\mathcal{H} = \pi\frac{\delta\mathcal{L}}{S\dot{\Phi}} - \mathcal{L} = \frac{1}{2}\{\pi^2 + (\nabla\Phi)^2 + m^2\Phi^2\} \tag{1.8}$$

where

$$\pi = \frac{\delta \mathcal{L}}{\delta \Phi} = \dot{\Phi} = \partial_0 \Phi$$

We use the natural units where $c \equiv 1$ and $\hbar \equiv 1$. So the action is dimensionless

$$[S] = m^0 \ ,$$

and for other quantities we get

$$\begin{aligned} [E] = [p] &= m \\ [x] &= m^{-1} \\ [\mathcal{L}] = [\mathcal{H}] &= m^4 \\ [\phi] &= m \end{aligned} \tag{1.9}$$

Exercise: prove the dimension rule eq. (1.9)

Equations of motion are derived from Hamilton variational principle

$$\begin{cases} \delta S = 0 \\ \partial_\mu \left(\frac{\delta \mathcal{L}}{\delta \partial_\mu \Phi} \right) = \frac{\delta \mathcal{L}}{\delta \Phi} \end{cases} \tag{1.10}$$

Eiler-Lagrange equations (1.10) for the density eq. (1.6) coincides with Klein-Gordon equation

$$(\partial^2 + m^2)\Phi = 0 \tag{1.11}$$

Consider the plane wave anzatz for the solution of eq. (1.11)

$$\Phi_{\vec{p}}(x,t) = a(t) e^{i\vec{p}\vec{x}} \tag{1.12}$$

The equation for the amplitude a

$$\ddot{a} + (\vec{p}^2 + m^2) a = 0 \tag{1.13}$$

is an equation for linear oscillator with frequency

$$\omega^2(p) = \vec{p}^2 + m^2 \ ,$$

or

$$\omega(p) = \pm\sqrt{p^2 + m^2}$$

We get that dependence of frequency ω on $\vec{p}$ and the dependence of particle energy on momentum $\vec{p}$ are exactly the same (in the units $\hbar = c = 1$). This is why we can use free fields to describe free particles.

The general solution in the periodic box can be presented as a superposition of the solutions (1.12)

$$\Phi(x) = \sum_p [a(p)e^{-ipx} + a^+(p)e^{ipx}] \tag{1.15}$$

where

$$px = p_\mu x_\mu = p_0 x_0 - \vec{p}\vec{x}$$

$$p_0 = \sqrt{\vec{p}^2 + m^2}$$

$$\sum_p = \int \frac{d^3p}{(2\pi)^3 2p_0}$$

In the Classical theory coefficient $a(\vec{p})$ are the arbitrary complex numbers. In terms of these variables the Hamiltonian is equal

$$H = \int d^3x \mathcal{H} = \sum_p \frac{1}{2}\omega(p)[aa^+ + a^+a]$$

$$\tag{1.16}$$

$$\omega(p) = \sqrt{\vec{p}^2 + m^2}$$

This is the Hamiltonian for the set of decoupled linear oscillators.

In Quantum Field Theory we have to quantize these oscillators. The variables $a(p)$ become operators that satisfy commutation relations

$$[a(\vec{p}), a^+(\vec{p}\,')] = \delta_{\vec{p}\vec{p}'}$$

$$\tag{1.17}$$

$$[a(\vec{p}), a(\vec{p}\,')] = [a^+(\vec{p}), a^+(\vec{p}\,')] = 0$$

Operators $a(p)$ and $a^+(p)$ are familiar from QM. They are the annihilation and creation operator for oscillator with frequency $\omega(\vec{p})$.

The Fock space is the Hilbert space of the states with definite values of the operator of particle number $N(p) = a^+(p)a(p)$:

$$\underline{\text{vacuum} \quad |0>}$$

$$\begin{cases} |0> \\ a(p)|0> \equiv 0 \end{cases},$$

<u>one-particle states</u>

$$|p>= a^+(p)|0> \tag{1.18}$$

<u>two-particle states</u>

$$|p_1, p_2>= a^+(p_1)a^+(p_2)|0>$$
$$|p; p>= \sqrt{2}a^+(p)a^+(p)|0>$$

etc.

We get that commutation relation eq. (1.17) corresponds to Bose-Einstein statistic for spin 0 particle

$$|p_1, p_2>= +|p_2, p_1>$$

and to positively defined operator of energy

$$H = \sum \omega(p)[N(p) + \frac{1}{2}] \tag{1.20}$$

The vacuum energy is equal

$$E_{vac} = \sum \frac{1}{2}\omega(p) \tag{1.21}$$

We have constructed the space of free particles with given momenta. Now we have to describe the propagation of free particles. The operator

$$\Phi^{(+)} = \sum_{\bar{p}} a(\bar{p})e^{-ipx}$$

with positive frequency is a combination of terms that annihilate 1 particle at point x. Operator

$$\Phi(x) = \sum_{\bar{p}} a^+(\bar{p})e^{ipx} \tag{1.22}$$

creates the particle at point x.

Consider the time ordering product

$$T\{\Phi(x), \Phi(0)\} = \Theta(x_0)\Phi(x)\Phi(0) + \Theta(-x_0)\Phi(0)\Phi(x) \tag{1.23}$$

where the step function is equal

$$\Theta(x) = \begin{cases} 1 & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x < 0 \end{cases}$$

So the Feynman propagator

$$D_F(x,0) = < 0|T\{\Phi(x)\Phi(0)\}|0 > \tag{1.24}$$

is the amplitude for a particle to propagate from point 0 to point x. Time ordering implies that creation always comes before annihilation.

The theory of the complex scalar fields $\Phi(x) = \frac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2)$ with lagrangian density

$$\mathcal{L} = (\partial_\mu \Phi^+ \partial_\mu \Phi) - m^2 \Phi^+ \Phi \tag{1.25}$$

is equal to the theory of two different scalar particles with degenerate masses. The general solution of the field equations can be presented in the form

$$\Phi(x) = \sum_p (a(p)e^{-ipx} + b(p)^+ e^{ipx}) \tag{1.26}$$

where the operator (a, a^+) and (b, b^+) are creation and annihilation operators for the particle with the same masses but with the opposite electric charges (see the next lecture). We consider these two particles as a particle and antiparticle.

The Feynman propagator of scalar particle in the momentum representation is equal to

$$\xrightarrow{\;\;p\;\;} \qquad\qquad D(p) = \frac{i}{p^2 - m^2 + i\varepsilon} \tag{1.27}$$

For Dirac spinor field $\Psi(x)$ the lagrangian density

$$\mathcal{L} = \bar{\Psi}[i\gamma_\mu \partial_\mu - m]\Psi \quad . \tag{1.28}$$

The plane wave solutions of the Dirac equation look like

$$u(p,\lambda)e^{ipx} \ , \quad (\lambda = \pm 1/2) \tag{1.29}$$

$$v(p,\lambda)e^{-ipx}\ , \quad (\lambda = \pm 1/2)$$

where $u(p,\lambda)$, $v(p,\lambda)$ satisfy equations

$$\begin{aligned}(\gamma_\mu p_\mu - m)u(p,\lambda) &= 0 \\ (\gamma_\mu p_\mu + m)v(p,\lambda) &= 0\end{aligned} \tag{1.30}$$

and $\lambda = \pm 1/2$ label the independent solution with different value of the spin projection on momenta $\vec{p}$. The general solution of Dirac equation can be presented in the form

$$\Psi(x) = \sum_{\vec{p},\lambda} \Big\{ a(p,\lambda)u(p,\lambda)e^{-ipx} + b^{+}(p,\lambda)v(p,\lambda)e^{ipx} \Big\} \tag{1.31}$$

where $a(p,\lambda)$ and $b(p,\lambda)$ are annihilation operators for particles and antiparticles respectively.

The next step is the quantization. We have to consider $a(p,\lambda)$ and $b(p,\lambda)$ as operators in the Fock space. The great surprise is that to have positively defined energy the operators $a(p,\lambda)$ and $b(p,\lambda)$ should satisfy <u>anticommutation</u> relations

$$\begin{aligned}\Big\{a(p,\lambda), a^{+}(p',\lambda')\Big\} = \Big\{b(p,\lambda), b^{+}(p',\lambda')\Big\} &= \delta_{pp'}\delta_{\lambda\lambda'} \\ \{a,a\} = \{a^{+},a^{+}\} = \{b,b\} = \{b^{+},b^{+}\} &= 0\end{aligned} \tag{1.32}$$

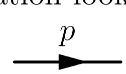
with $\{A,B\} = AB + BA$.

These imply Fermi-Dirac statistic for spin 1/2 particle. These two examples demonstrate the famous spin-statistic theorem.

Feynman propagator $S_F(x)$

$$S_F = < 0|T\{\Psi(x)\bar{\Psi}(y)\}|0 > \tag{1.33}$$

in momentum representation looks like



$$S(p) = \frac{i}{\hat{p} - m} \tag{1.34}$$

where $\hat{p} = \gamma_\mu p_\mu$.

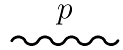
Exercise. Calculate the dimension of field Ψ: $[\Psi] = m^{3/2}$.

Electromagnetic Field $A_\mu(x)$:
Lagrangian density is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + e j_\mu^{ext} A_\mu \tag{1.35}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Because of gauge invariance the quantization of electromagnetic field is rather subtle matter. In Feynman gauge the propagator for photon



$$D_F^{\mu\nu} = \frac{(-i) g_{\mu\nu}}{p^2 + i\varepsilon} \tag{1.36}$$

Exercise. Show that

$$[A_\mu] = m$$

$$[j_\mu] = m^3$$

1.3. Feynman Rules and Feynman Amplitudes. Tree approximation.

What we do understand well is the QFT in the framework of perturbation theory. Free field theory provides the asymptotic $|\text{in}>$ and $|\text{out}>$ states for free particles and the amplitudes for propagation of free particles from one space-time point to another point. The nonlinear interaction term $\mathcal{L}_{int}$ in perturbation theory provides the vertices. Combining vertices and propagators one can construct in perturbation theory the transition amplitude from one asymptotic state to another one. Let us remind the main steps.

Transitions are described by means of unitary S-matrix: $S^+ S = I$

$$< f|S|i > = < f|i > + (2\pi)^4 i \delta^{(4)}(\Sigma p_f - \Sigma p_i) < f|T|i > \tag{1.37}$$

where i and f refer to initial and final state.

In perturbation theory

$$S = Texp\{i \int d^4x \mathcal{L}_{int}\} =$$

$$= I + i \int d^4x \mathcal{L}_{int}(x) + \frac{i^2}{2} T\{\int d^4x_1 \mathcal{L}(x_1); \int d^4x_2 \mathcal{L}(x_2)\} + ... \tag{1.38}$$

where the operators of fields are in the <u>interaction</u> representation.

Consider for example the case of QED. The interaction looks like a product of electromagnetic current and 4-potential

$$\mathcal{L}_{int} = j_\mu^{em}(x) A_\mu(x)$$

$$j_\mu^{em}(x) = (-ie)\{\bar{e}(x)\gamma_\mu e(x) - \frac{2}{3}\bar{u}(x)\gamma_\mu u(x) + ...\} \tag{1.39}$$

Feynman rules for this QED lagrangian are summarized in Fig. 1.

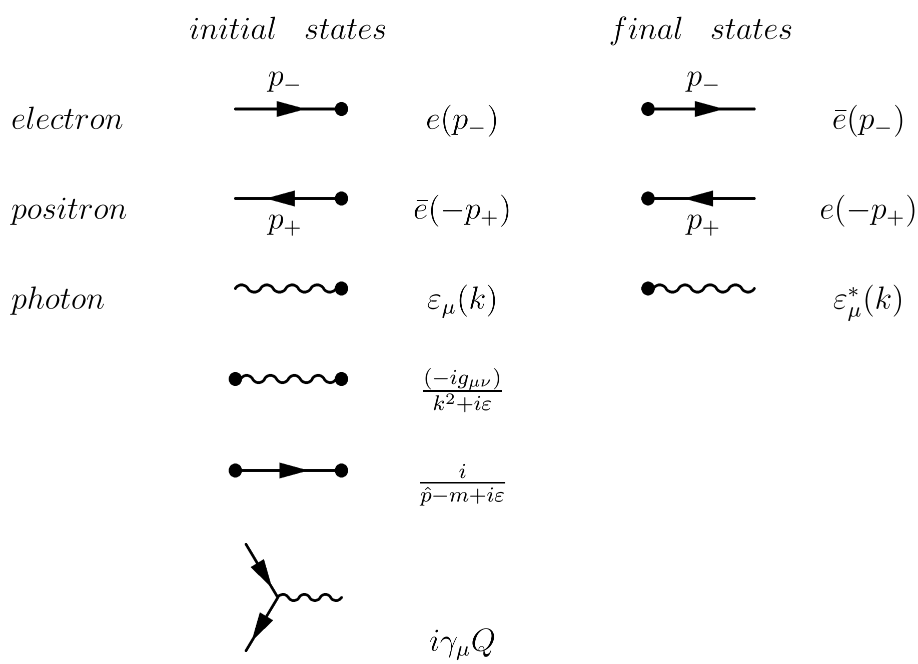
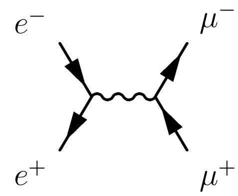


Figure 1: Feynman rules of QED.

Using these rules one can easily construct the transition amplitude. Consider as an example the process $e^+e^- \to \mu^+\mu^-$. There is one diagram for this process



The amplitude T is equal

$$iT(e^+e^- \to \mu^+\mu^-) = (-ie)^2 j_\alpha^{(e)} \frac{(-ig_{\alpha\beta})}{q^2} j_\beta^{(\mu)} \tag{1.40}$$

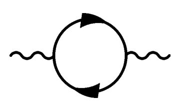
where

$$j_\alpha^{(e)} = \bar{e}(-p_+)\gamma_\alpha e(p_-)$$
$$j_\beta^{(\mu)} = \bar{\mu}(k_-)\gamma_\beta \mu(-k_+)$$

This is the example of the amplitude in the lowest order in the coupling constant e. It contains no loops. There is special name for such diagrams – tree diagrams.

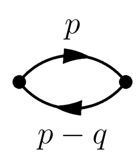
1.4. Loop corrections. Propagator corrections in QED.

Consider one-loop correction to the photon propagator



$$\delta D_{\mu\nu} = \frac{(-ig_{\mu\alpha})}{q^2}(-i\Pi_{\alpha\beta}(q))\frac{(-i)g_{\beta\nu}}{q^2} \tag{1.41}$$

where

$(-i)\Pi_{\alpha\beta}(q) \equiv \alpha$  $\beta =$

$$= e^2 \int \frac{d^4p}{(2\pi)^4}(-1)Sp\gamma_\alpha \frac{1}{\hat{p} - m + i\varepsilon}\gamma_\beta \frac{1}{\hat{p} - \hat{q} - m + i\varepsilon} \tag{1.42}$$

For large virtual momenta p the loop correction diverges quadratically

$$\Pi_{\alpha\beta} \simeq e^2 g_{\alpha\beta} \int_0^\Lambda \frac{d^4p}{p^2} \simeq g_{\alpha\beta} e^2 \Lambda^2 \to \infty \tag{1.43}$$

for $\Lambda \to \infty$.

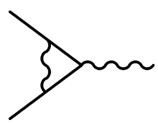
The one-loop correction to electron propagator

is proportional to

$$e^2 \frac{\hat{p}}{p^2 - m^2} \int \frac{d^4 q}{q^4} \sim e^2 \frac{\hat{p}}{p^2 - m^2} \ln \Lambda^2 \to \infty \tag{1.44}$$

for $\Lambda \to \infty$

The vertex



is proportional to

$$e^3 \gamma_\mu \ln \Lambda^2 \to \infty \tag{1.45}$$

for $\Lambda \to \infty$.

These corrections diverge logarithmically.

These are the simplest examples of the problem of divergences in QFT. It was a great success of theoretical physics when Dyson, Feynman, Schwinger and Tomanaga in the late 40th explained how to work with such theories.

1.5. Renormalizable Field Theories QED.

The general philosophy of renormalization can be formulated in the following way:

1) Suppose that we can split quantum fluctuations on the "fast" fluctuations (i.e. with virtual momenta $p > \Lambda$) and on the "slow" ones ($p < \Lambda$), where Λ is arbitrary large parameter.

2) Suppose that we can integrate over the "fast" fluctuations even though the physics at small distances ($p > \Lambda$) can be unknown.

3) For "slow" fluctuations we get "effective field theory" with $\mathcal{L}^{eff}(\Lambda)$ or $S^{eff}(\Lambda)$ and with parameters that depend on cut-off Λ.

4) Physics of the low-energy processes does not depend on the value of the Λ.

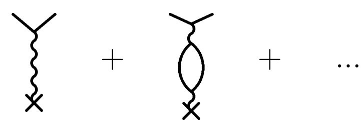
5) For special class of renormalizable theories $S^{eff}(\Lambda)$ depends on finite number of parameters and interaction terms.

Consider how this program works in the case of QED. Suppose that effective lagrangian has the form

$$\mathcal{L}(\Lambda) = -\frac{1}{4}(F^B_{\mu\nu})^2 + \bar{\Psi}_B(i\gamma_\mu\partial_\mu - m_B)\Psi_B -$$

$$-e_B\bar{\Psi}_B\gamma_\mu\Psi_B A^B_\mu \tag{1.46}$$

where all quantities with label B depend on Λ. Consider the scattering of heavy charged particle on the Coulomb center. In this case we have to sum up all corrections to the photon propagator.



As a result at low-energy the amplitude of Coulomb scattering is equal to

$$T = \frac{e^2_B(\Lambda)}{1 - \frac{e^2_B(\Lambda)}{12\pi^2}\ln\frac{\Lambda^2}{m_e^2}} \cdot \frac{1}{q^2} \tag{1.47}$$

The coefficient in front of $1/q^2$ is by definition the charge of particle ($1/q^2$ corresponds to $1/r$ dependence in the Coulomb law). So we claim that combination

$$e^2_{ph} = \frac{e^2_B(\Lambda)}{1 - \frac{e^2_B(\Lambda)}{12\pi^2}\ln\frac{\Lambda^2}{m_e^2}} \tag{1.48}$$

is the physical charge. It does not depend on Λ and in this way we find $e^2_B(\Lambda)$ as a function of Λ.

In the similar way one can define the physical electron mass $m_{ph} = m_e$ as a pole in the exact propagator of the electron.

Now we are able to formulate the main theorem.

Theorem. If we rewrite the amplitudes of all QED processes that depend on e_B, m_B and Λ in terms of e_{ph}, m_{ph} the dependence on Λ in these amplitudes will disappear for large Λ!

1.6. Non-renormalizable Theories.

Fermi Theory (1934)

The first theory of weak interactions was formulated by Fermi. It was very similar to QED. The lagrangian of interaction was equal to a product of two vector currents. After the discovery of P and C parity violation this 4-fermion theory was modified so that

$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} j_\alpha j_\alpha \tag{1.49}$$

where $j_\alpha = \bar{\nu}_e \gamma_\alpha (1+\gamma_5) e + ...$

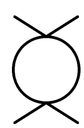
Remember that

$$[j] = m^3 \; ; \;\; [\mathcal{L}] = m^4$$

so the Fermi coupling constant has dimension -2:

$$[G_F] = m^{-2}$$

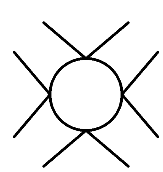
From the dimensional analysis it is clear that radiative corrections to 4-fermion interaction



should be of the order of

$$G_F(1 + \Sigma(G_F \Lambda^2)^n)(j)^2$$

where Λ is cut-off. It is also clear that 4-fermion interaction can generate multi-fermion interaction with diverge coupling constant, e.g. 8-fermion interaction



$$\Delta\mathcal{L}^{eff} = CG_F^4[\ln\Lambda^2 + \Sigma(G_F\Lambda^2)^n](j)^4$$

etc.

In this way we find that $\mathcal{L}^{eff}$ should depend on infinite number of terms.

This is the example of non-renormalizable theory. In such theories we have to fix infinite number of terms in $\mathcal{L}^{eff}(\Lambda)$ at the scale Λ (i.e. at small distances $x \sim \Lambda^{-1}$) to reconstruct the amplitudes at low energy. Nobody knows how to work with nonrenormalizable theories.

1.7. From field theory to cross sections.

The steps from the formal QFT to the numerical predictions for cross sections and for the decay rates are very simple

$$\begin{gathered}\text{Feynman diagrams}\\ \Downarrow\\ \text{Amplitude } T\\ \Downarrow\\ \text{Probability} \sim \Sigma|T|^2\end{gathered}$$

For example the Cross Sections are calculated by the formula

$$d\sigma_{fi} = \frac{1}{2\sqrt{\lambda(s,m_1^2,m_2^2)}}|T_{fi}|^2 d\tau$$

where

$$d\tau = (2\pi)^4\delta^4(p_1+p_2-\Sigma p_f)\prod_{j=1}^{N}\frac{d^3p_j}{(2\pi)^3 2E_j}$$

is N-particle phase space and

$$\lambda(s,m_1^2,m_2^2) = 4[(p_1p_2)^2 - m_1^2m_2^2]$$

is relativistic flux.

The Decay Rate is given by formula

$$d\Gamma = \frac{1}{2E}|T|^2 d\tau$$

There exists a well developed routine technology of this kind of calculations. But sometime we can understand the basic properties of the answer

without long calculations. Consider three examples of the order of magnitude estimates

1. Decay $\mu \to e\tilde{\nu}_e\nu_\mu$

In this case we know that

$$[\Gamma] = m \ \ [G_F] = m^{-2} \ ;$$

and that in the limit $m_\mu \gg m_e$ nothing should depends on m_e. So on the ground of the dimensional analysis we conclude

$$\Rightarrow \Gamma(\mu \to e\tilde{\nu}_e\nu_\mu) = \frac{1}{192\pi^3} G_F^2 m_\mu^5$$

Certainly the numerical factor is beyond of these order of magnitude estimates but the dependence on mass is understood well.

2.Cross section $\nu e \to \nu e$

Cross sections have dimension

$$[\sigma] = m^{-2}$$

So for very large energy when we can forget about masses

$$\sigma(\nu_e e \to \nu_e e) = \frac{1}{3\pi} G_F^2 s$$

3.Cross section $e^+e^- \to \mu^+\mu^-$

Fine coupling constant is dimensionless. The cross section does not depend on masses at high energy. So

$$\sigma = \frac{4}{3}\frac{\pi\alpha^2}{s} \ , \ \ s \gg m^2$$

It is interesting to note that after some training one can restore the powers of π that originate from the phase space. We have no time for such training.

Lecture II. **Symmetries.**

In the Standard Model the notion of global and local symmetries plays the very important role. In this lecture we shall study the different aspects of symmetries in QFT using a well known physical examples.

2.1. Global symmetries.

We start with the simplest $U(1)$ symmetry and consider as an example the theory of free electrons. Electrons are described by 4-component complex field $\psi_i(i = 1,2,3,4)$ named bispinor. The free Lagrangian density has the form

$$\mathcal{L} = \bar{\psi}(x)(i\gamma_\mu \partial_\mu - m)\psi(x) \tag{2.1}$$

where $\bar{\psi} = \psi^+ \gamma_0$ and γ_μ are 4×4 Dirac matrices, m is the electron mass.

It is quite evident that $U(1)$ global phase transformations

$$\begin{array}{lcl} \psi(x) & \rightarrow & \psi'(x) = e^{i\alpha}\psi(x) \\ \bar{\psi}(x) & \rightarrow & \bar{\psi}'(x) = \bar{\psi}(x)e^{-i\alpha} \end{array} \tag{2.2}$$

leave Lagrangian (2.1) invariant. The global symmetry means that the phase of the transformation is the same for any of space-time points x.

A little bit less trivial example is the theory of two complex self-interacting scalar fields with the degenerate masses

$$\mathcal{L} = \partial_\mu \Phi^+ \partial_\mu \Phi - m^2 \Phi^+ \Phi - \frac{\lambda}{4}(\Phi^+\Phi)^2, \tag{2.3}$$

where Φ is the two component column (doublet)

$$\Phi = \begin{pmatrix} \varphi^+(x) \\ \varphi^0(x) \end{pmatrix} \tag{2.4}$$

The Lagrangian (2.3) is invariant under global $SU(2)$ rotations of the complex doublet Φ

$$\begin{array}{lcl} \Phi(x) & \rightarrow & \Phi'(x) = S\Phi(x) \\ \Phi^+(x) & \rightarrow & (\Phi'(x))^+ = \Phi^+(x)S^+ \end{array} \tag{2.5}$$

where S is unitary 2×2 matrix

$$S^+S = I$$

$$\text{with} \quad detS = 1 \tag{2.6}$$

This matrix S can be represented in the form

$$S = \exp[i\frac{\tau_a}{2}\alpha^a] \tag{2.7}$$

where $\tau_a = (\tau_1, \tau_2, \tau_3)$ are Pauli matrices

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \ \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \ \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.8}$$

that satisfy to $SU(2)$ Lie algebra commutation relations

$$[\tau_i \ , \ \tau_k] = 2ie_{ik\ell}\tau_\ell \tag{2.9}$$

where $e_{ik\ell}$ is totally antisymmetric tensor with $e_{123} = 1$. Three independent phases $\alpha^a(a = 1, 2, 3)$ do not depend on space-time. The Lagrangian density (2.3) describes Higgs scalars in the Standard Model.

Another useful example is the approximate $SU(2)_L \times SU(2)_R$ symmetry of strong interactions. This symmetry has been discovered long before the formulation of QCD in the framework of such general approach as the current algebra. It provided the first example of nonlinear realization of symmetry in QFT. It this lecture we shall formulate this symmetry using the Lagrangian of QCD.

The mass scale of QCD is defined by $\Lambda_{\rm QCD}$:

$$\Lambda_{\rm QCD} \simeq 0.5 \text{ Gev} \tag{2.10}$$

On the other hand according to particle data group booklet the masses of up and down quarks are of the order of a few Mev. So in a good approximation one can take

$$m_{u,d} = 0 \tag{2.11}$$

In this limit QCD Lagrangian for u and d quarks can be rewritten as

$$\mathcal{L} = \bar{\psi}_L(i\gamma_\mu\mathcal{D}_\mu)\psi_L + \bar{\psi}_R(i\gamma_\mu\mathcal{D}_\mu)\psi_R \tag{2.12}$$

where $\psi(x) = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}$ is $SU(2)$ doublet constructed from bispinor $u(x)$ and $d(x)$. The subscribes L and R means left and right field by definition

$$
\begin{array}{rcl}
u_{L,R} & = & \frac{1}{2}(1 \pm \gamma_5)u \\
d_{L,R} & = & \frac{1}{2}(1 \pm \gamma_5)d
\end{array}
\tag{2.13}
$$

where $\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$.

Any bispinor $\psi(x)$ can be represented as a sum of two spinors (left and right Weyls spinors)

$$
\psi = \psi_L + \psi_R = \frac{1}{2}(1 + \gamma_5)\psi + \frac{1}{2}(1 - \gamma_5)\psi \tag{2.14}
$$

For the case of massless particles the Lagrangian itself can be written as a sum of two independent Lagrangians : one for left spinors, another for right spinors. Each of them is $SU(2)$ invariant. So the total symmetry is $SU(2)_L \otimes SU(2)_R$.

2.2. Noether's Theorem. Conserved Currents.

So far we considered the examples of different Lagrangians that were invariant with respect to $U(1)$ and $SU(2)$ transformations. The transformations were global, i.e. they did not depend on space-time points x. As for invariance of the Lagrangian it looked like rather trivial property.

Noether's theorem states that for any continuous global symmetry of the Lagrangian one can construct the conserved vector currents. This is dynamical statement and it does not look like being trivial at all. I am going to prove this theorem in the classical field theory.

Let Lagrangian L depends on the set of fields φ^i and its first derivatives $\varphi^i_{,\mu} = \partial_\mu \varphi^i$. For infinitesimal global transformations the variations of fields are equal to

$$
\begin{array}{rcl}
\delta\varphi^i & = & i\epsilon^{(a)} T^{(a)}_{ij} \varphi^j \\
\delta\varphi^i_{,\mu} & = & i\epsilon^{(a)} \partial_\mu (T^a_{ij} \varphi^j)
\end{array}
\tag{2.15}
$$

The real infinitesimal parameters $\epsilon^{(a)}$ represent the independent symmetry transformations, matrices T^a_{ij} are the generators of the group of transformations in given representations. The invariance means that the actions S is not changed under transformation (2.15):

$$
\delta S = \int d^4x \delta L \equiv 0 \tag{2.16}
$$

Let us calculate the variation of Lagrangian density directly

$$
\begin{aligned}
\delta L &= \tfrac{\partial L}{\partial \varphi^i}\delta\varphi^i + \tfrac{\partial L}{\partial \varphi^i_{,\mu}}\delta\varphi^i_{,\mu} = \\
&= +[\partial_\mu \tfrac{\partial L}{\partial \varphi^i_{,\mu}}]\delta\varphi^i + \tfrac{\partial L}{\partial \varphi^i_{,\mu}}\delta\varphi^i_{,\mu}
\end{aligned}
\qquad (2.17)
$$

where we have used the Lagrangian equation of motion for $\varphi^i(x)$

$$
\frac{\partial L}{\partial \varphi^i} = \partial_\mu \frac{\partial L}{\partial \varphi^i_{,\mu}} \qquad (2.18)
$$

Substituting the variations for $\delta\varphi$ and $\delta\varphi_{,\mu}$ from eqs. (2.15) into (2.17) and (2.16) we get

$$
\delta S = i\epsilon^{(a)} \int d^4x \partial_\mu j^a_\mu(x) = 0 \ , \qquad (2.19)
$$

where

$$
j^{(a)}_\mu = \frac{\partial L}{\partial \varphi^i_{,\mu}} T^{(a)}_{ij} \varphi^j \qquad (2.20)
$$

Therefore we get the conservation of Noether currents

$$
\partial_\mu j^{(a)}_\mu(x) = 0 \qquad (2.21)
$$

and the conservation of the corresponding charges

$$
\frac{d}{dt} Q^{(a)}(t) = 0 \qquad (2.22)
$$

$$
Q^{(a)}(t) = \int d^3x j^{(a)}_0(x) \qquad (2.23)
$$

(We assume that there are no fields at spatial infinity, i.e. $\int d^3x \partial_i j_i \equiv 0$)

The generalization of this proof to the Quantum Field Theory requires more advanced techniques such as operators algebra, commutators etc. But the final results, i.e. the expression for conserved Noethers currents, remain the same. So we are going to skip the proof of the theorem in QFT.

At the end of this subsection we present the conserved vector currents that correspond to the symmetries that we considered in the subsection 2.1 :

$$
\begin{aligned}
U(1) &: \ j_\mu = \bar{\psi}\gamma_\mu\psi \ , \\
SU(2) &: \ j^a_\mu = \Phi^+ \tau^a \overleftrightarrow{\partial_\mu} \Phi
\end{aligned}
\qquad (2.24)
$$

$SU(2)_L \times SU(2)_R$:

$$
(j^{L,R}_\mu)^a = \bar{\psi}_{L,R} \gamma_\mu \tau^a \psi_{L,R}
$$

where

$$\Phi^+ \overleftrightarrow{\partial_\mu} \Phi = \Phi^+ \partial_\mu \Phi - (\partial_\mu \Phi^+) \phi$$

2.3. Spontaneous Violation of Global Symmetry. Goldstone Phenomenon.

The idea of spontaneous violation of symmetry was formulated first in the solid state physics. Consider, for example, the piece of some ferromagnetic material. The interaction of the elementary magnetic moments of electrons inside ferromagnetic is $O(3)$ invariant. On the other hand at low temperature $T < T_c$ the total magnetic moment $\overrightarrow{M}$ of ferromagnetic piece is nonzero and has the definite direction, i.e. it violates $O(3)$ invariance of the system. Ground state is only $O(2)$ invariant for the rotations around $\overrightarrow{M}$ and the "violated" symmetries are realized as a massless excitations.

In field theory analogous phenomenon is known as Nambu-Goldstone realization of symmetry. We are going to discuss this phenomenon using the example of $SU(2)_L \times SU(2)_R$ symmetry of strong interactions. Quark Lagrangian (2.12) is $SU(2)_L \times SU(2)_R$ invariant. There are 3 left $(V - A)$ and 3 right $(V + A)$ conserved currents of massless quarks. In other words there are 3 vector and 3 axial vector conserved currents. This is evident. On the other hand the quarks do not exist like a free particles. Instead we have a set of massive hadrons – baryons and mesons. The $SU(2)_V$ symmetry of hadrons was known for a long time. For example proton and neutron have practically degenerate mass and can be treated as an up and down members of $SU(2)_V$ doublet $N = \binom{p}{n}$. (There are small corrections to the $SU(2)$ symmetric approximation of the order of $(\frac{m_{u,d}}{\Lambda})^2$ and of the order of electroweak coupling constant α.) The matrix elements of the conserved $SU(2)$ vector currents between nucleon states have the form

$$< N|\bar{\psi}\gamma_\mu \tau^a \psi|N > \sim \bar{N}\gamma_\mu \tau^a N \tag{2.25}$$

In the limit of degenerate mass this m.e. is transversal

$$q_\mu \bar{N}\gamma_\mu \tau^a N = 0 \quad , \tag{2.26}$$

i.e. it corresponds to conserved currents.

For conservation it is crucial to have baryons with <u>degenerate</u> masses. The symmetry is realized in such a way that it transforms one-particle baryonic state into another one-particle state with the same mass.

If this realization of symmetry is unique we immediately get troubles with the $SU(2)_A$ symmetry. Indeed to construct the transversal matrix element of axial current we need degenerate baryons with opposite P-parity. The brief search for such baryons in the Table of Particle Properties shows that such baryons do not exist in Nature. So this way is prohibited for $SU(2)_A$.

One may try another possibility for matrix element of axial current between the same nucleon states

$$< N|\bar{\psi}\gamma_\mu\gamma_5\tau^a|N >\sim (g_{\mu\nu} - \frac{q_\mu q_\nu}{q})\bar{N}\gamma_\nu\gamma_5\tau^a N \ , \tag{2.27}$$

where q is momentum transfer from one nucleon to another. The transversality is now evident because

$$q_\mu(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) \equiv 0$$

So we have solved this problem easily. But now matrix element (2.27) is singular. It has a pole at $q^2 = 0$. The poles in the amplitudes correspond to one particle intermediate states and pole at $q^2 = 0$ corresponds to massless particle. Fortunately there are π-mesons that contribute into m.e. (2.27) and that are almost massless. Now we are able to formulate the new way of realization of approximate $SU(2)_A$.

According to this new philosophy π-mesons should be massless in the limit $m_{u,d} = 0$ and there should be simple relation between the axial-vector part of matrix element

$$g_{\mu\nu}\bar{N}\gamma_\mu\gamma_5 N \tag{2.28}$$

and the pseudoscalar part

$$-\frac{q_\mu q_\nu}{q^2}\bar{N}\gamma_\mu\gamma_5 N = -\frac{q_\mu}{q^2}2m_N(\bar{N}\gamma_5 N) \tag{2.29}$$

to have conserved current. This relation is known as Goldberger-Treiman relation. The matrix element of axial current between nucleon states can be measured in $n \to pe\tilde{\nu}$ decay. Experimental value of the ratio of axial coupling constant to the pseudoscalar coupling constant is very close to the theoretical prediction. So this realization of symmetry indeed works in the case of $SU(2)_A$.

Instead of one-particle degenerate states with opposite P-parity we have massless pseudoscalar Goldstone particles - pions. The symmetry transforms one-particle baryonic state into degenerate two-particle state (baryon plus

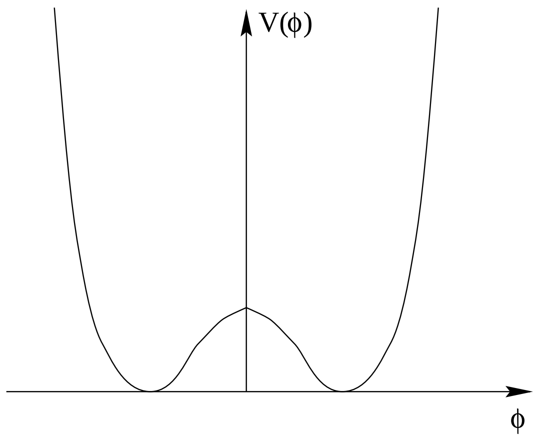


Figure 1: Higgs Potential in Standard Model

massless pseudoscalar pion), then into 3-particle state with two pions etc. This is nonlinear realization of symmetry. One can proceed further and show that there are infinitely many states with the same lowest energy. The vacuum state is one of these states and it violates $SU(2)_A$ symmetry. This violation is due to nonzero vacuum expectation value of quark condensate

$$\begin{aligned} < 0|\bar{u}u|0 > &= < 0|\bar{d}d|0 > \neq 0 \\ < 0|\bar{u}\gamma_5 u|0 > &= < 0|\bar{d}\gamma_5 d|0 > = 0 \end{aligned} \tag{2.30}$$

It seems more instructive not to spend time proving eqs. (2.30) but to consider the same phenomenon using a very simple field model studied many years ago by Goldstone.

Let us start with the theory of complex scalar field $\varphi(x)$ with Lagrangian

$$\mathcal{L} = \partial_\mu \varphi^+ \partial_\mu \varphi - V(|\varphi|^2) \tag{2.31}$$

and with a special choice of potential (see fig.1)

$$V(|\varphi|^2) = \lambda(|\varphi|^2 - \frac{\eta^2}{2})^2 \tag{2.32}$$

Lagrangian (2.31) is invariant under $U(1)$ transformations

$$\varphi(x) \to \varphi'(x) = e^{i\Lambda}\varphi(x) \tag{2.33}$$

and the Noether current is

$$j_\mu = i\varphi^+ \overleftrightarrow{\partial_\mu} \varphi \tag{2.34}$$

There are continuously many minima of the potential V (2.32)

$$\varphi = \frac{1}{\sqrt{2}}\eta e^{i\alpha} \tag{2.35}$$

The vacuum corresponds to one of these minima. This is spontaneous violation of symmetry : we have chosen one of state as a vacuum from the infinite set of minima. Let vacuum state corresponds to zero phase $\alpha = 0$:

$$\varphi_{cl} = \frac{1}{\sqrt{2}}\eta \tag{2.36}$$

Consider the small fluctuation of fields near vacuum configuration

$$\varphi = \frac{1}{\sqrt{2}}[\eta + \rho(x) + i\sigma(x)] \tag{2.37}$$

Potential can be rewritten as

$$V(\varphi) = V(\rho, \sigma) = \frac{\lambda}{2}\left\{(\sigma^2 + \rho^2)^2 + 4\eta\rho(\rho^2 + \sigma^2) + 4\eta^2\rho^2\right\} \tag{2.38}$$

The coefficients in front of bilinear terms determine the mass of the fields. So we get a theory of two particles with masses

$$\begin{matrix} M_\rho^2 = 4\lambda\eta^2 \\ M_\sigma^2 \equiv 0 \end{matrix} \tag{2.39}$$

Excitations that correspond to the motion along the valley of minima are massless! This is Goldstone phenomenon.

We can use more elegant and transparent representation for $\varphi(x)$ to demonstrate this phenomenon. Let us rewrite $\varphi(x)$ in terms of modulus and phase

$$\varphi(x) = \rho(x)e^{i\sigma(x)} \tag{2.40}$$

Then

$$\mathcal{L}(\rho, \sigma) = (\partial_\mu\rho)^2 - V(\rho^2) + \rho^2(\partial_\mu\sigma)^2 \tag{2.41}$$

There is no dependence on the field σ in the potential and therefore this field corresponds to massless particle.

In Quantum Field Theory we have two ways for realization of symmetry:

1) Vacuum state has the symmetry of the action S. Excitation states are degenerate.

2) Vacuum state has lower symmetry than action S. There are flat direction in configuration space of fields. The motions along these flat directions correspond to massless Goldstone particles.

Exercise

Consider the double well potential in Quantum Mechanics

$$V(x) = \lambda(x^2 - \eta^2)^2$$

and in the Quantum Field Theory for real scalar field $\varphi(x)$;

$$V(\varphi) = \lambda(\varphi^2 - \eta^2) \ .$$

Show that in QM there is only one lowers state and in QFT there are two degenerate and orthogonal lowest states.

2.4. Local U(1) gauge symmetry.

Now we are going to study the new type of symmetries : local gauge symmetries. Let us start with the theory of complex field $\varphi(x)$ described by the Lagrangian (eq.(2.31))

$$\mathcal{L} = \partial_\mu \phi^+ \partial_\mu \phi - V(\phi^+ \phi)$$

invariant under global $U(1)$ transformation

$$\phi(x) \to \phi'(x) = e^{i\Lambda} \phi(x) \ .$$

Consider now the local $U(1)$ transformation when we change the phase of the field independently for any point x

$$\phi(x) \to \phi'(x) = e^{i\Lambda(x)} \phi(x) \qquad (2.42)$$

The potential $V(|\phi|^2) = V(|\phi'|^2)$ is invariant under this transformation but the kinetic term is not

$$\partial_\mu \phi^+ \partial_\mu \phi \to |(\partial_\mu + i(\partial_\mu \Lambda)\phi|^2 \tag{2.43}$$

To compensate this non-invariant change one can introduce new vector field $A_\mu(x)$ with the transformation law :

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu \Lambda(x) \tag{2.44}$$

so that the new Lagrangian

$$\mathcal{L} = |(\partial_\mu - ieA_\mu)\phi|^2 - V(|\phi|^2) \tag{2.45}$$

is locally $U(1)$ invariant or gauge invariant. The combination $\mathcal{D}_\mu = \partial_\mu - ieA_\mu$ has a name of covariant derivative (or long derivative). It has a simple transformations law

$$\begin{array}{rcl} D_\mu & \to & e^{i\Lambda}\mathcal{D}_\mu e^{-i\Lambda} \\ D_\mu\phi & \to & e^{i\Lambda}(\mathcal{D}_\mu\phi) \end{array} \tag{2.46}$$

Up to now the fields $A_\mu(x)$ have no kinetic term in the Lagrangian and they are some kind of the auxiliary fields that do not propagate. To construct kinetic term we need gauge invariant combination of the derivatives of field A_μ. Notice that covariant derivatives and any combinations of the covariant derivatives have a very simple transformation law eq. (2.46). Consider the commutator of two derivatives,

$$\begin{array}{l} [\mathcal{D}_\mu \mathcal{D}_\nu] \equiv -ieF_{\mu\nu} \\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \end{array} \tag{2.47}$$

We see that commutator is not the the differential operator but the function of x. According to (2.46) it is gauge invariant function. Now we are in position to write the totally gauge invariant Lagrangian

$$\begin{array}{rcl} \mathcal{L} & = & -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + |\mathcal{D}_\mu\phi|^2 - V(|\phi|^2) \\ \phi(x) & \to & \phi'(x) = e^{i\Lambda(x)}\phi(x) \\ A_\mu(x) & \to & A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\Lambda(x) \end{array} \tag{2.48}$$

The notion of gauge invariance was introduced by V. Fock in 1926 and by H. Weyl in 1929. (The very interesting history of this subject can be found in the lectures given by L.Okun at this school ten years ago).

2.5. Spontaneous Violation of local symmetry. Higgs Phenomenon.

For the case when time derivative is zero $D_o\phi = 0$ and electric field is zero $F_{oi} = E_i = 0$ the Lagrangian (2.48) formally is equal to the free energy in the Ginzburg-Landau phenomenological theory of superconductivity, where $\varphi(x)$ plays a role of the order parameter. It is known that magnetic field does not penetrate into superconductor, it falls exponentially. Exponential fall in QFT corresponds to a massive particle. So one can expect that Lagrangian (2.48) at certain circumstances can describe the massive gauge field. This is the famous Higgs mechanism of spontaneous violation of local symmetry.

Consider Lagrangian (2.48) with the special choice of potential energy (2.32)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |\mathcal{D}_\mu\phi|^2 - \lambda(|\phi|^2 - \frac{\eta^2}{2})^2 \tag{2.49}$$

Potential $V(\phi)$ has continuous valley of minima. Let us quantize the fields near the vacuum state (2.32)

$$< \varphi >= \varphi_{cl} = \frac{1}{\sqrt{2}}\eta \tag{2.50}$$

As in the case of global symmetry it is convenient to use representation of $\phi(x)$ in term of modulus and phase

$$\phi(x) = \frac{1}{\sqrt{2}}(\eta + \rho(x))e^{i\sigma(x)} \tag{2.51}$$

The Lagrangian (2.48)-(2.49) is gauge invariant. So let us make gauge transformation with $\Lambda(x) \equiv -\sigma(x)$

$$\begin{array}{rcl} \phi(x) & \rightarrow & \phi' = e^{i\sigma}\phi \\ A_\mu(x) & \rightarrow & A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\sigma \end{array} \tag{2.52}$$

In this gauge (unitary gauge)

$$D_\mu\phi \rightarrow (\partial_\mu - ieA_\mu)\frac{1}{\sqrt{2}}(\eta + \rho(x)) \tag{2.53}$$

and the Lagrangian can be rewritten in the form

$$\begin{aligned}\mathcal{L} \;=\; & \left[-\tfrac{1}{4}F_{\mu\nu}^2 + \tfrac{1}{2}e^2\eta^2 A_\mu^2\right] + \\ & \tfrac{1}{2}(\partial_\mu\rho)^2 + (e^2\eta)\rho(x)A_\mu^2(x) + \tfrac{e^2}{2}A_\mu^2(x)\rho^2(x)\end{aligned} \tag{2.54}$$

The term in bracket represents the free massive vector particle with mass

$$m_V = e\eta \tag{2.55}$$

Massless Goldstone mode $\sigma(x)$ has been eaten by massless vector field $A_\mu(x)$ (that had two polarization) and as a result we get massive vector field with three polarization. This is Higgs Phenomenon.

2.6. Local $SU(2)$. Yang-Mills theory of vector fields.

We have to make another nontrivial step to be ready for the construction of the Standard Model. We have to consider the general case of the local gauge groups.

Let us start with $SU(2)$ theory of massless fermion $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$

$$\mathcal{L} = \bar{\psi}[i\gamma_\mu\partial_\mu\psi] \tag{2.56}$$

and consider local $SU(2)$ transformations

$$\psi(x) \to \psi'(x) = S(x)\psi(x) \tag{2.57}$$

where

$$\begin{aligned}&S(x) = \exp i(T_j\Lambda_j(x)) \;; \\ &T_i = \tfrac{1}{2}\tau_i \quad , \quad i = 1,2,3 \;; \\ &[T_i \;,\; T_j] = ie_{ijk}T_k\end{aligned} \tag{2.58}$$

The Lagrangian (2.56) is not invariant under this transformation. To compensate the non-invariant piece in the Lagrangian we introduce the triplet of vector fields $A_\mu^i(x)$ so that:

$$\begin{aligned}&\mathcal{L} = \bar{\psi}i\gamma_\mu(\partial_\mu - igA_\mu(x))\psi \\ &A_\mu(x) = T^i A^i(x)\end{aligned} \tag{2.59}$$

with the transformation law

$$A_\mu(x) \to A'_\mu(x) = SA_\mu(x)S^+ - \frac{i}{g}(\partial_\mu S)S^+ \tag{2.60}$$

Again it is convenient to introduce covariant derivative

$$\mathcal{D}_\mu = \partial_\mu - igA_\mu \tag{2.61}$$

that transforms as a triplet under $SU(2)$ transformations:

$$\begin{array}{ll} D_\mu \to & SD_\mu S^+ \\ D_\mu\psi \to & S(D_\mu\psi) \end{array} \tag{2.62}$$

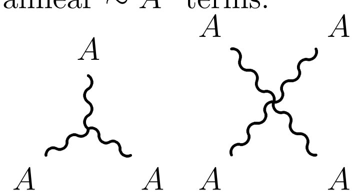
We can define the triplet of field-strength tensor $G^i_{\mu\nu}$:

$$\begin{array}{ll} G_{\mu\nu} & \equiv G^i_{\mu\nu}T^i = \frac{i}{g}[\mathcal{D}_\mu, \mathcal{D}_\nu] \\ & = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu A_\nu] \\ & G_{\mu\nu} \to G'_{\mu\nu} = SG_{\mu\nu}S^+ \end{array} \tag{2.63}$$

and construct the $SU(2)$ gauge invariant Lagrangian

$$\mathcal{L} = -\frac{1}{4}Tr[G_{\mu\nu}G_{\mu\nu}] + \bar{\psi}i\gamma_\mu\mathcal{D}_\mu\psi \tag{2.64}$$

This Lagrangian was invented by Yang and Mills in 1954. The very nontrivial part in this construction is that kinetic energy $\sim G^2_{\mu\nu}$ contains bilinear $\sim A^2$, trilinear $\sim A^3$ and quadralinear $\sim A^4$ terms:



So we have a gauge theory of self-interacting vector fields.

2.7. Spontaneous Violation of Local $SU(2)$ Symmetry. Renormalizable theory of massive vector fields.

Consider the $SU(2)$ gauge theory of the couple of scalar fields $\phi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$:

$$\mathcal{L} = -\frac{1}{4}TrG_{\mu\nu}G_{\mu\nu} + |\mathcal{D}_\mu\phi|^2 - \lambda(|\varphi|^2 - \frac{\eta^2}{2})^2 \tag{2.65}$$

We expect that after spontaneous violation of $SU(2)$ symmetry three Goldstone bosons will be mixed with three massless vector fields and produce

three massive vector filed. To show this we repeat the steps that we had done in the case of local $U(1)$ symmetry.

Let us introduce a special representation for the doublet ϕ

$$\phi(x) = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = e^{i\sigma^i(x)T^i} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(\eta + \rho(x)) \end{pmatrix} \tag{2.66}$$

and consider gauge transformation with the parameter

$$\Lambda^i(x) = -\sigma^i(x) \tag{2.67}$$

In this gauge the fields $\sigma^i(x)$ disappear from the Lagrangian and vector part of $\mathcal{L}$ gets the form

$$\begin{cases} \mathcal{L}_{\text{vect}} = -\frac{1}{4}TrG^2_{\mu\nu} - \frac{1}{2}m_V^2 A_\mu^2 \\ m_v = \frac{1}{2}g\eta \end{cases} \tag{2.68}$$

This is the theory of massive vector fields with the special choice of self-interactions.

The theory of massless Yang-Mills field was renormalizable theory. It seems that the property of the vacuum should not change the behavior of the amplitudes at high energy. So one can believe that Yang-Mills theory with spontaneous violation of gauge symmetry remains renormalizable. The theory of massive vector fields with arbitrary interactions is nonrenormalizable in general. But if one takes the special case of interaction with quarks, with scalars and self-interaction that corresponds to the gauge-invariant Lagrangian (2.65) the nonrenormalizable divergences should disappear. Technically the rigorous proof of this statement is quite nontrivial business even now. This problem had been solved by t'Hooft and Veltman in 1971.

Lecture III. **SU(2)×U(1) Theory of Electroweak Interactions.**

In this lecture I am going to describe the fundamental Lagrangian of the Standard Model. It is important to understand that **a priori** there is no unique way to construct the model of electroweak interactions. There are plenty of them. In the review paper by B.Bjorken and Llewellyn-Smith in 1973 they discussed several dozens of models. We do not understand yet why the gauge group is SU(2)×U(1), why there are three generations of quarks and leptons etc. We have to deduce our theory from the experiment.

3.1. Minimal group.

It was well established in old four-fermionic theory of weak interactions that charged currents (responsible for β-decay of nucleons and other hadrons) have $V-A$ structure, i.e. they are constructed from the left-handed fermions.

The minimal group of gauge symmetry which includes charged vector currents is $SU(2)$ group. So any theory of weak interactions have to include $SU(2)_L$ symmetry as a subgroup. Photon interacts both with left- and right-handed fermions. So if we are going to unify weak and electromagnetic interactions the group of gauge symmetry should include $U(1)$ as well. The simplest choice of such group of symmetry is

$$G = SU(2)_L \times U(1)$$

3.2. Left and Right Fermions. $SU(2)_L$ symmetry. Weak Mixing Angles.

Any Dirac 4-spinor Ψ can be presented as a sum of two Weyl spinors Ψ_L and Ψ_R:

$$\Psi = \Psi_L \oplus \Psi_R = \frac{1}{2}(1+\gamma_5)\Psi + \frac{1}{2}(1-\gamma_5)\Psi \tag{3.1}$$

These Weyl spinors are irreducible representation of Lorentz group, they depend on two complex parameters. For massless fermions

$$\Psi_L = \begin{pmatrix} (1-\vec{\sigma}\vec{n})\varphi \\ -(1-\vec{\sigma}\vec{n})\varphi \end{pmatrix} \quad , \tag{3.2.}$$

where φ is 2-spinor, $\vec{\sigma}$ are Pauli matrices, and $\vec{n} = \vec{p}/|p|$ is the direction of the motion of particle. So for left particle Ψ_L

$$\vec{\sigma}\vec{n} = -1 \tag{3.3}$$

and for right particles Ψ_R

$$\vec{\sigma}\vec{n} = +1 \tag{3.4}$$

Left leptons and quarks group into $SU(2)_L$ doublets. For the first generations they are

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \text{and} \quad Q = \begin{pmatrix} u \\ d \end{pmatrix}_L \tag{3.5}$$

To avoid $V + A$ charged current we have to put right fermions into singlet representation. So e_R, u_R and d_R are singlets. As for right-handed neutrino ν_R nobody has observed it so far. It is unknown whether such field exists. Just now we prefer not to introduce ν_R into the theory.

To include the electromagnetic interactions we have to define charge. For left-handed fermions the charge is different for up and down component so that

$$Q_L = T_3 + Y_L \tag{3.6}$$

where T_3 is the third component of $SU(2)_L$ and Y_L is left hypercharge.

From eq.(3.6) it follows that for leptonic doublet $Y_L = -1/2$ and for quark doublet $Y_Q = 1/6$.

For right fermions we identify Q and Y_R:

$$Q_R = Y_R \tag{3.7}$$

so that $Y_{u_R} = \frac{2}{3}$; $Y_{d_R} = -\frac{1}{3}$, $Y_{e_R} = -1$.

The minimal way to introduce $U(1)$ interactions is to consider gauge boson that interacts with

$$Y = Y_L + Y_R \tag{3.8}$$

This is the gauge group of Minimal Standard Model

$$SU(2)_L \times U(1)_Y \quad . \tag{3.9}$$

Let $A^i_\mu(x)$, $i = 1, 2, 3$ be gauge bosons of $SU(2)_L$ and $B_\mu(x)$ – the gauge boson of $U(1)$ group. The charged fields

$$A^{\pm}_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \pm iA^2_\mu) \tag{3.10}$$

can be identify with $W^{\pm}_\mu$ bosons.

Photon $A_\mu(x)$ is in general some combination of A^3_μ and B_μ. Orthogonal combination represents another physical particle that we identify with Z boson. So

$$\left(\begin{array}{c} Z_\mu \\ A_\mu \end{array}\right) = \left(\begin{array}{cc} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & cos\theta_W \end{array}\right)\left(\begin{array}{c} A^3_\mu \\ B_\mu \end{array}\right) \tag{3.11}$$

where θ_W is a weak mixing angle.

To violate spontaneously $SU(2)_L \times U(1)_Y$ group and to make masses to $W^\pm$ and Z bosons we need three Goldstone fields. The $SU(2)$ doublet of Higgs particles

$$H = \left(\begin{array}{c} H^+ \\ H^0 \end{array}\right) \;;\quad Y_H = \frac{1}{2} \tag{3.12}$$

can provide this number of Goldstone bosons after spontaneous violation. In the MSM we use only **one** Higgs doublet.

We have completed the construction of the MSM. Now we are ready to calculate the masses of vector bosons m_W, m_Z and phenomenological mixing angle θ_W in terms of coupling constants g_2 for $SU(2)$, g_1 for $U(1)$ and in terms of v.e.v. of Higgs field η.

In the unitary gauge Higgs doublet has the form

$$H(x) = e^{i\vec{T}\vec{\alpha}(x)}\left(\begin{array}{c} 0 \\ \frac{1}{\sqrt{2}}(\eta + \rho(x)) \end{array}\right) \tag{3.13}$$

Covariant derivative

$$D_\mu \equiv \partial_\mu - ig_1 Y B_\mu(x) - ig_2 T^a A^a_\mu(x) \tag{3.14}$$

for the vacuum field H_{vac}

$$\begin{aligned} D_\mu H_{vac} &= (-ig_1\frac{1}{2}B_\mu - ig_2\frac{1}{2}\tau^a A^a_\mu)\left(\begin{array}{c} 0 \\ \frac{\eta}{\sqrt{2}} \end{array}\right) = \\ &= \frac{(-i)}{2\sqrt{2}}\eta\left(\begin{array}{c} \sqrt{2}g_2 W^-_\mu \\ -g_2 A^3_\mu + g_1 B_\mu \end{array}\right) \end{aligned} \tag{3.15}$$

The mass term for vector fields originates from $(D_\mu H)^+ D_\mu H$ term in the Lagrangian. It looks like

$$\mathcal{L}_{mass} = \frac{1}{4}(g_2\eta)^2 W^+_\mu W^-_\mu + \frac{1}{8}\eta^2(g_2 A^3_\mu - g_1 B_\mu)^2 \tag{3.16}$$

From this expression we conclude that the massive combination of A_μ^3 and B_μ (i.e. Z-boson) is

$$Z_\mu = \frac{1}{\sqrt{g_1^2+g_2^2}}(g_2 A_\mu^3 - g_1 B_\mu) \tag{3.17}$$

or that

$$tg\theta_W = g_1/g_2 \tag{3.18}$$

From eq. (3.16) it follows that

$$m_W = \frac{1}{2} g_2 \eta \tag{3.19}$$

and

$$m_W = m_Z \cos\theta_W \tag{3.20}$$

It is very interesting that Z boson should be heavier than W boson! After spontaneous violation there still remains unbroken $U(1)$ symmetry that corresponds to massless photon.

If we introduce electric charge e as a coupling constant of the photon we can relate $g_{1,2}$ with e and $\cos\theta_W$. Let us rewrite interaction of A_μ^3 and B_μ as an interaction of A_μ and Z_μ fields:

$$(-ig_2T_3)A_\mu^3 - ig_1 Y B_\mu \equiv (-i)\frac{g_2}{\cos\theta_W}[T_3 - \sin^2\theta_W Q]Z_\mu + (-i)(g_1\cos\theta_W)QA_\mu \tag{3.21}$$

This is identically rewritten universal expression for covariant derivative. So eq. (3.21) is applicable to the left and right fermions and to the Higgs doublet.

From eq. (3.21) it follows immediately that

$$e = g_1 \cos\theta_W = g_2 \sin\theta_W \tag{3.22}$$

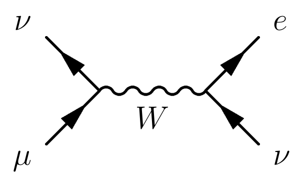
We complete the description of bosonic sector of the SM.

3.3. Weak interactions of leptons and quarks. Neutral Current. Request for new particles.

Now we are ready to calculate the amplitude for the first physical process, for the decay of $\mu \to e\nu\tilde{\nu}$. Charged currents Lagrangian for leptons looks like

$$\Delta\mathcal{L}_{Charged} = \frac{g_2}{2\sqrt{2}} W_\mu^+[\bar{\nu}\gamma_\mu(1+\gamma_5)e + ...] \tag{3.23}$$

where the dots are for the similar terms with μ and τ leptons. Feynman diagram for the μ-decay is presented in Fig.1



The amplitude for the decay can be read from this diagram and it is equal to

$$T(\mu \to e\nu\tilde{\nu}) = [\frac{g_2}{2\sqrt{2}}]^2 \frac{1}{m_W^2 - q^2} (\bar{\nu}\gamma_\alpha(1+\gamma_5)\mu)(\bar{e}\gamma_\alpha(1+\gamma_5)\nu) \qquad (3.24)$$

The momentum transfer q from muonic current to electronic current is of the order of muonic mass m_μ. So if $m_W \gg m_\mu$ the amplitude looks like a point-like interaction in Fermi theory.

$$T_{Fermi} = \frac{G_F}{\sqrt{2}} j_\alpha^e (j_\alpha^\mu)^+ \qquad (3.25)$$

Comparing these two presentations for the same amplitude we conclude that

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8m_W^2} \qquad (3.26)$$

Taking into account eq. (3.19) for m_W we also get that v.e.v. η is directly connected with G_F:

$$\eta = [\sqrt{2}G_F]^{-1/2} = 246 \text{ GeV} \qquad (3.27)$$

$$G_F \equiv G_\mu = 1.16639(2) \cdot 10^{-5} \text{ GeV}^{-2}$$

To fix remaining two fundamental parameters g_1 and g_2 we have to choose two other physical observables measured with the best accuracy. The choice is evident. They are the fine coupling constant α

$$\alpha^{-1} = \frac{4\pi}{e^2} = 137.035985(61) \qquad (3.28)$$

and the mass of Z-boson

$$m_Z = 91.187(2) \text{ GeV} \qquad (3.29)$$

To calculate g_1 and g_2 we first have to calculate the mixing angle θ_W in terms of G_F, α and m_Z. It is not difficult exercise to show that

$$\sin^2\theta_W \cos^2\theta_W = \frac{\pi\alpha}{\sqrt{2}(G_F m_Z^2)} \quad . \tag{3.30}$$

Exercise 1: Derive eq. (3.30).

Substituting the values of the parameters from eqs. (3.27), (3.28) and (3.30) we get

$$\begin{aligned} \sin^2\theta_W &= 0.2120 \\ g_1 &= \frac{\sqrt{4\pi\alpha}}{\cos\theta_W} = 0.34 \\ g_2 &= \frac{\sqrt{4\pi\alpha}}{\sin\theta} = 0.66 \end{aligned} \tag{3.31}$$

So we are ready for the first prediction in SM: we can calculate m_W

$$(m_W)^{theor} = m_Z \cos\theta_W = 80.94 \text{ GeV} \tag{3.32}$$

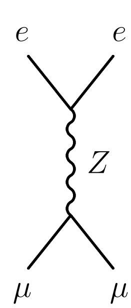
that has to be compared with the current experimental value

$$(m_W)^{exp} = 80.37(8) \text{ GeV} \tag{3.33}$$

The deviation from theoretical number is only 0.6%, but this tiny number is equal to 8σ deviation. To explain the huge discrepancy we have to take into account radiative correction that have the scale of the few per mill.

The old 4-fermionic point-like theory is the effective theory for momentum transfer much smaller than m_W. In this sense the SM is generalization of the old theory. But SM also predicts the new phenomena that were unknown in V-A theory. This is the neutral currents.

The effective 4-fermionic coupling of neutral currents is generated by Z boson exchange.



At small momentum transfer it is local interaction with the coupling constant equal to $G_F \cos^2\theta_W$.

Exercise 2. Calculate the coupling constant for neutral currents.

Though this coupling is of the same order as G_F by some reasons the experimental search for neutral currents gave negative results for a long time and only in 1973 experimental groups at CERN observed neutral currents and provided the first experimental measurements of $\cos\theta_W$. This measurement gave the possibility to calculate m_W and m_Z theoretically (eqs. (3.19), (3.20)) with rather good accuracy. This estimate had been extremely helpful for the experimental discovery of W and Z bosons.

Another great achievement of the SM was the request for new particles needed for self-consistency of the theory. In 1970 the set of the known particles included 2 generations of leptons

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \,, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \;; \quad e_R \,, \mu_R \tag{3.34}$$

and three quarks u, d and s that belong to the following $SU(2)_L \times U(1)_Y$ representation

$$\begin{pmatrix} u \\ d' = d\cos\theta_c + s\sin\theta_c \end{pmatrix}_L \,, \quad u_R \,, \quad d_R \,, \quad s_R \tag{3.35}$$

where θ_c is the Cabibbo angle. First of all there was no symmetry between quarks and leptons. No less important was the fact that for this set of quarks Z boson exchange produces flavour-changing $s \leftrightarrow d$ neutral currents

$$Z_\mu \bar{d}'_L \gamma_\mu d'_L \sim Z_\mu \Big[(\bar{d}d)\cos^2\theta_c + (\bar{s}s)\sin^2\theta_c + \\ + \sin\theta_c\cos\theta_c[\bar{d}s + \bar{s}d] \Big] \tag{3.36}$$

This was absolutely forbidden by experimental data. To save the SM Glashow, Illiopoulous and Maiani in 1970 introduced fourth c quark and the new $SU(2)_L$ doublet

$$\begin{pmatrix} c \\ -d\sin\theta_c + s\cos\theta_c \end{pmatrix} \tag{3.37}$$

As a result flavour-changing neutral currents disappear and all neutral currents become diagonal. This theoretical request for new particle was satisfied by experimental discovery of c-quark in 1974.

3.4. Quark masses. CKM matrix and CP-violation.

In the Standard Model the standard mass term for the electron violates $SU(2)_L$. Indeed this term

$$m_e \bar{e} e = m_e [\bar{e}_R e_L + \bar{e}_L e_R] \tag{3.38}$$

transforms like doublet instead of being invariant.

To preserve $SU(2)_L \times U(1)_Y$ symmetry we have to use Higgs mechanism to generate the masses for fermions. For example Yukawa coupling of L, e_R and H is $SU(2)_L \times U(1)_Y$ invariant

$$\begin{aligned} \Delta\mathcal{L} &= f_e(\bar{L} e_R) H + h.c. = \\ &= \frac{f_e}{\sqrt{2}}(\eta + \rho(x))\bar{e}e = \\ &= m_e \bar{e} e + \frac{f_e}{\sqrt{2}}\rho(x)\bar{e}e \end{aligned} \tag{3.39}$$

where $\rho(x)$ is the field for physical Higgs in SM. From eq. (3.39) it follows that Yukawa coupling is proportional to m_e

$$f_e = \frac{\sqrt{2}}{\eta} m_e \simeq 3 \cdot 10^{-6} \tag{3.40}$$

Notice that before this step the fields $e_L(x)$ and $e_R(x)$ were absolutely different, i.e. they had different interaction with W and Z. Yukawa interaction unified this two Weyl spinors into one massive particle – electron. To give the mass to down quarks we can use the same type of Yukawa interaction

$$\Delta\mathcal{L}_{m_d} = f_d(\bar{Q}_L d_R) H \tag{3.41}$$

As for the mass of up quarks we need Higgs doublet with nonzero v.e.v. for up component of doublet. At that moment we can introduce new Higgs doublet. But in the case of $SU(2)$ group complex conjugated fields

$$\tilde{H} = (-i\sigma_2) H^* \tag{3.42}$$

also behave like a member of $SU(2)$ doublet. So we can use $\tilde{H}$ to give mass to upper quark

$$\Delta\mathcal{L}_m = f_d(\bar{Q}_L d_R) H + f_u(\bar{Q}_L u_R)\tilde{H} \tag{3.43}$$

This is the solution of problem of fermion mass in the case of one generation. For more than one generation we have to take into account quark mixing.

Cabibbo-Kobayashi-Maskawa (CKM) matrix.

For three generations of quarks the general Yukawa couplings produce general non-diagonal mass 3×3 matrix

$$\Delta\mathcal{L} = (\bar{u}'_L)_i M^{(u)}_{ik} (u'_R)_k + (\bar{d}'_L)_i M^{(d)}_{ik} (d'_R)_k + h.c. \tag{3.44}$$

where u'_i, d'_i are quarks that belong to $SU(2)$ doublet of i-th generation $i = 1, 2, 3$.

The matrices $M^{(u)}$ and $M^{(d)}$ can be diagonalized by use of left and right unitary rotation, i.e.

$$\begin{aligned} M^{(u)} &= U_L^+ M^{(u)}_{diag} U_R \\ M^{(d)} &= D_L^+ M^{(d)}_{diag} D_R \end{aligned} \tag{3.45}$$

where $M^{(u,d)}_{diag}$ are diagonal matrices. Substituting this expression into eq. (3.44) we get that diagonal massive quark fields are

$$\begin{aligned} u_R = U_R u'_R \ , \ \ u_L = U_L u'_L \ ; \\ d_R = D_R d'_R \ , \ \ d_L = D_L d'_L \ ; \end{aligned} \tag{3.46}$$

It means that the members of the $SU(2)$ doublet are the mixture of different massive fields. We already met such doublet in the case of 2 generations (see eqs. (3.35) and (3.37)).

In terms of massive fields the charged currents look like

$$j^+_\mu = \bar{u}_L \gamma_\mu d'_R = \bar{u}_L (U_L D_L^+) \gamma_\mu d_L \tag{3.47}$$

The unitary 3×3 matrix

$$V = U_L D_L^+ \tag{3.48}$$

is the famous CKM matrix.

Due to unitarity of U and D matrices the neutral currents remain diagonal

$$\bar{u}'_R u'_R = \bar{u}_R u_R \ , \ \ \bar{u}'_L u'_L = \bar{u}_L u_L \ , \text{etc.} \tag{3.49}$$

You will have a course of lectures on CKM matrix by Y.Nir. I do not want to interfere with his lectures but I'd like to mention one fundamental property of CKM. In general case $n \times n$ unitary matrix can be represented as the product of different orthogonal rotations dependent on n_a angles

$$n_a = \frac{1}{2} n(n-1) \tag{3.50}$$

and of the $U(1)$ factors with n_{ph} observable phases

$$n_{ph} = \frac{1}{2}(n-1)(n-2) \tag{3.51}$$

and of a number of nonobservable $U(1)$ factors that can be absorbed into redefinition of quark's fields. So for three generation there is one observable phase, i.e. charge currents contain complex couplings. This immediately gives violation of CP-invariance. Till now it is unknown whether this is the only source of CP-violation in the Nature. But in any case CKM mechanism of CP-violation does exist.

3.5. Subtle point. Triangle Anomaly.

To have renormalizable theory of electroweak interactions it was absolutely crucial to start from the gauge invariant theory where gauge bosons interact with conserved Noether currents. Spontaneous violation of symmetry does not spoil any symmetric relations between operators. They are exactly the same as in non-violent theory. The confusing notion of spontaneous violation means only nonlinear realization of the symmetry in the space of physical states.

In the SM we operate both with vector and axial currents. For any axial currents

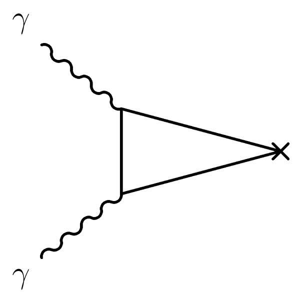
$$\begin{gathered} j^A_\mu = \bar{\Psi}\gamma_\mu\gamma_5\Psi \\ \partial_\mu j^A_\mu = 2im\bar{\Psi}\gamma_5\Psi \end{gathered} \tag{3.52}$$

So naively for massless fermion axial current is conserved. But what is true in Classical Field Theory can be not true in Quantum Field Theory. Indeed one-loop calculation of the divergence of axial current for electrons gives

instead of eq. (3.52)

$$
\begin{cases}
\partial_\mu j^5_\mu = 2im\bar{\Psi}_e\gamma_5\Psi_e + \frac{\alpha}{2\pi}F_{\mu\nu}\tilde{F}_{\mu\nu} \\
\\
\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}
\end{cases}
\tag{3.53}
$$

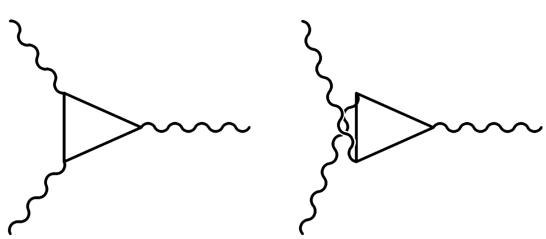
The term $F\tilde{F}$ originates from matrix element of $\partial_\mu j^5_\mu$ between vacuum and two-photon states.



So the axial current is not conserved even for $m \equiv 0$. Not any classical symmetry can survive in Quantum Mechanics. This very interesting theoretical phenomenon has special name – triangle anomaly.

In the SM there are plenty of axial currents that interact with gauge fields. Though fermions are massless (no mass terms in the Lagrangian) the anomaly can destroy the conservation of Noether currents and this will kill renormalizability. There is one possibility to save it. We see from eq. (3.53) that anomaly depends only on the "charge" of particle that is running inside loop. So if the total gauge current has different pieces it can happen that nonzero individual anomalies cancel each other for total current.

This cancellation imposes some restrictions on the charges of quarks and leptons. Let us check this possibility. We will calculate the triangle matrix elements between fields A^i_μ and B_μ. There are two crossing diagrams that contribute to anomalous interaction between 3 gauge fields.



Consider first the anomalous contribution of one generation of matter. It is easy to see that:

1) (A, A, A) and (A, B, B) anomalies are automatically disappeared for lepton doublet and for quark doublet separately.

2) (B, A, A) anomaly is disappeared if

$$Q_e + 2Q_u + Q_d \equiv 0 \quad , \tag{3.54}$$

i.e. quark contribution cancels lepton contribution only for this special relation between charges. This relation means that hydrogen atom has to be neutral!

It is very interesting that renormalizability of the SM takes place only if the charge of proton is opposite to the charge of electron.

We can proceed further and consider other anomalies. At that moment we have to make some statement about ν_R. Suppose first that it does not exist at all. In this case:

3) Cancellation of (B, B, B) anomaly takes place only if

$$Q_e = -1 \ , \ \ Q_\nu = 0 \ ; \ \ Q_u = \frac{2}{3} \ , \ \ Q_d = -\frac{1}{3} \tag{3.55}$$

(We suppose that QCD has $SU(3)_c$ symmetry.)

4) Cancellation of ($B \to$ gluon + gluon) anomaly is automatic.

5) Cancellation of ($B \to$ graviton + graviton) anomaly takes place only for the charge sample eq. (3.55). So we are able to fix the relative charges of leptons and quarks in this case.

If ν_R does exist anomalies 3) - 5) are disappeared automatically for any charge of neutrino.

Exercise IV. Prove 1) - 5).

If we suppose that the new generations are the exact replica of the old one (only masses are different, but the charges are the same) then we come to the same conclusion for each generation. If we allow to change the charges from generation to generation the restrictions on the choice of charges becomes weaker. But in any case it is very interesting that renormalizability impose restrictions on the property of matter fields.

Lecture IV. **Higgs, $\boldsymbol{W}$ and $\boldsymbol{Z}$.**

Higgs boson H is the only missing particle in the Standard Model. The interaction of H with gauge bosons Z, W and photon γ is fixed by gauge invariance of the SM. There is no freedom in this interaction.

As for the Yukawa coupling constants of H with quarks and leptons they are free parameters of the theory. Yukawa coupling constants determine the masses of quarks and leptons, the mixing CKM angles and CP-odd phase. We do not understand yet why these parameters of SM have given value and not another one. So we have to take it from the experimental data.

A little can be said about Higgs particle. In this lecture we are going to discuss this isoteric subject.

4.1. Higgs Sector. Custodial symmetry of Higgs Potential.

The Higgs potential in the SM

$$V(H) = \frac{\lambda}{4}(H^+H - \frac{\eta^2}{2})^2 \tag{4.1}$$

has been constructed to be $SU(2) \times U(1)$ invariant. Here $H = \binom{H^+}{H^0}$ is $SU(2)$ doublet with $Y_L = 1/2$, H^+H is singlet. So $SU(2) \times U(1)$ symmetry is absolutely evident.

In the unitary representation

$$H(x) = exp[i\alpha_a(x)\frac{\tau_a}{2}] \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(\eta + \rho(x)) \end{pmatrix} \tag{4.2}$$

$SU(2) \times U(1)$ transformations change three Goldstone fields $\alpha_a(x)$ and do not change the modulus $\rho(x)$. Potential $V(H)$ in this representation depends only on the modulus $\rho(x)$:

$$V(H) = V(\rho) = \frac{1}{2}(\frac{1}{2}\lambda\eta^2)\rho^2(x) + \frac{1}{4}\lambda\eta\rho^3 + \frac{1}{16}\lambda\rho^4 \tag{4.3}$$

Potential (4.1) has minima at nonzero value of H. Quantization near one of such minima

$$< H >_{vac} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \eta \end{pmatrix} \tag{4.4}$$

spontaneously violates $SU(2) \times U(1)$ symmetry up to the $U(1)$ symmetry that remains unbroken.

In the SM the v.e.v. η is connected with Fermi coupling constant G_F (see previous lecture) and is equal numerically to

$$\eta = 246 \text{ GeV} \ .$$

In the unitary representation potential $V(\rho)$ eq. (4.3) has 3 terms: quadratic, cubic and quartic ones. The quadratic term determines the mass of Higgs boson

$$m_H^2 = \frac{1}{2}\lambda\eta^2 \tag{4.5}$$

and cubic and quartic terms determine self-interaction of Higgs bosons.

In general theory the mass and the self-interaction are independent parameters but not in the SM. Potential (4.1) depends only on two parameters η and λ. So the Higgs boson mass completely fix the strength of self-interaction. For example the lower bound for m_H from LEP II experiment

$$m_H \gtrsim 90 \text{ GeV} \tag{4.6}$$

can be immediately transformed into lower bound for self-interaction coupling

$$\lambda = \frac{2m_H^2}{\eta^2} \gtrsim 0.27 \tag{4.7}$$

For large m_H the interaction of Higgs bosons becomes getting strong $\lambda \gtrsim 1$. In this case one should worry about large corrections to the low-energy observables, say to the ratio m_W/m_Z.

Naively one can expect that for $\lambda \sim 1$ the corrections are of the order of unity and that in this case the SM looses its predictive power. Fortunately this naive expectation is not correct and corrections to electroweak observables at low-energy (i.e. to LEP observables) are small even for heavy Higgs bosons. The reason is very subtle and beautiful. There is hidden symmetry of Higgs that preserves low-energy observables from large corrections. This is custodial $SU(2) \times SU(2)$ symmetry. Potential (4.1) that looks like $SU(2) \times U(1)$ invariant possesses larger $SU(2) \times SU(2)$ symmetry.

To find this symmetry it is convenient to rewrite (4.1) using new representation for doublet H:

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = (\pi_0 + i\pi_a\tau_a)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \pi_2 + i\pi_1 \\ \pi_0 - i\pi_3 \end{pmatrix} \tag{4.8}$$

where π_0 and π_a are real scalar fields. At first sight we have done nothing but changed a little bit the notation for H^+ and H^0. It is true, but in this new notation H^+H looks like a scalar product of four-vector $\pi_\mu = (\pi_0, \pi_a)$. Indeed

$$H^+H = (0,1)[\pi_0 - i\vec{\pi}\vec{\tau}][\pi_0 + i\vec{\pi}\vec{\tau}] \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \pi_0^2 + \vec{\pi}^2 = \pi_\mu \pi_\mu \quad . \tag{4.9}$$

The quadratic form $\pi_\mu \pi_\mu = \pi_0^2 + \vec{\pi}^2$ is invariant with respect to $O(4)$ orthogonal transformation and $O(4)$ group can be presented as

$$O(4) = SU(2) \times SU(2) \quad . \tag{4.10}$$

So for one Higgs doublet H any $SU(2) \times U(1)$ invariant potential $V(H^+H)$ is $SU(2) \times SU(2)$ invariant as well.

After spontaneous violation of symmetry the field π_0 gets nonzero v.e.v. and surviving symmetry is $O(3)$ rotational symmetry between different components of π_i or as a rotational symmetry between Goldstone fields α_i:

$$\alpha_i(x) \rightarrow \alpha_i'(x) = O_{ij} \alpha_j(x) \tag{4.11}$$

This $O(3)$ symmetry is custodial symmetry of Higgs potential.

The immediate consequence of the existence of the custodial symmetry is that the relation between m_W and m_Z

$$m_W / m_Z = \cos\theta \tag{4.12}$$

is correct to any order of λ. On other words there are no corrections of the order of λ^2, λ^3 etc. that could charge the prediction (4.12) by order of magnitude for $\lambda \gtrsim 1$. Why?

If we start the calculation of the corrections of the order of λ^2, λ^3 etc. to the effective action we immediately find such large corrections to the v.e.v. η and to the wave function renormalization Z of Goldstone fields α_i. (Wave function renormalization of gauge fields and gauge coupling constants are proportional to the square of gauge coupling constant and is considered as the small corrections.) Due to $O(3)$ custodial symmetry (4.11) of the selfinteraction the renormalization of the Goldstone fields should be the same for each of the components $\alpha_i(x)$, so that renormalized fields $\alpha^R(x)$ are transformed as a vector under $O(3)$ custodial symmetry

$$\vec{\alpha}^R(x) = Z^{1/2} \vec{\alpha}^B(x) \tag{4.13}$$

where $\vec{\alpha}^B$ are bare Goldstone fields. This is the central point. Indeed since $\vec{\alpha}^B(x)$ is a $O(3)$ vector it can be eaten by gauge transformation. In this gauge (unitary gauge) the renormalized effective action looks like the bare action that we considered in section 3.1. The only difference is that the Higgs vertices, the Higgs field $\rho(x)$ and v.e.v. η are renormalized by self-interaction in the order of λ^1, λ^2 ... So in this approximation we get that

$$\begin{aligned} m_W^R &= (\pi\alpha_W)\eta^R \\ m_Z^R &= (\pi\alpha_Z)\eta^R \end{aligned} \tag{4.14}$$

(see eqs. (3.19) and (3.20)) and that

$$m_W^R/m_Z^R = \alpha_W/\alpha_Z = \cos\theta_W$$

This relation is valid in any order of λ!

If we switch on the coupling constants g_2 and g_1 and consider the corrections of the order of α, $\alpha\lambda$, $\alpha\lambda^2$, ... the custodial $SU(2) \times SU(2)$ symmetry will be broken and we can expect the correction of the same order α_1, $\alpha\lambda$, $\alpha\lambda^2$, ... to the ratio m_W/m_Z and to other observables. These corrections do exist. They are of the order of two-loop electroweak corrections if the Higgs boson mass is not much larger than $m_{W,Z}$

$$\alpha_W \cdot \lambda \sim \alpha_W^2 \left(\frac{m_H}{m_Z}\right)^2$$

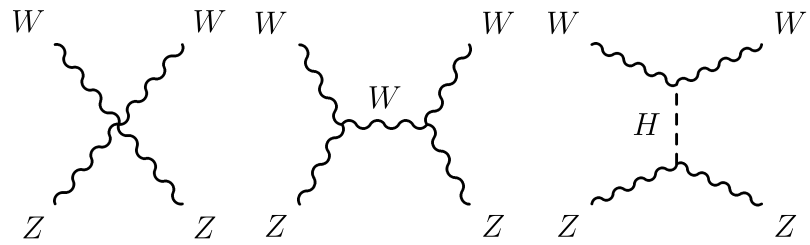
These small corrections will be studied in the next lecture.

4.2. Unitary Bound on Higgs mass.

Coupling constant λ of selfinteraction and the mass m_H^2 are independent parameters in the general theory but in the SM they are proportional to each other

$$m_H = \sqrt{\frac{\lambda}{2}} \cdot \eta \simeq \sqrt{\frac{\lambda}{2}}(246 \text{ GeV}) \tag{4.15}$$

If $m_H \gg m_{W,Z}$ the $SU(2)\times U(1)$ gauge theory looks like the old-fashioned theory of massive vector particles that is nonrenormalizable one. In nonrenormalizable theories scattering amplitudes grow with energy and violate unitary bounds at some value of energy. Consider for example the scattering of W boson on Z boson. There are three types of diagrams that contribute into this process.



$1a$ $1b$ $1c$

It is clear that for $m_H \to \infty$ the diagrams (1c) goes to zero as $1/m_H^2$ and the remaining two diagrams (1a,b) are the same as in the theory of massive W and Z without Higgs mechanism.

We are going to demonstrate that these two amplitudes for longitudinally polarized W_L and Z_L linearly grow with s, where s is square of the energy in the c.m.f. Indeed any polarization vector of massive vector particle satisfies the condition

$$\varepsilon_\mu^V(p) p_\mu = 0 \tag{4.16}$$

For momenta $p_\mu = (\varepsilon, 0, 0, p)$ the solution of eq. (4.16) with longitudinal components is

$$\begin{aligned} \varepsilon_\mu^V(p) &= \frac{1}{m_V}(p, 0, 0, \varepsilon) \simeq \\ &\simeq \frac{P_\mu}{m_V} + 0(\frac{m_W}{\varepsilon}) \ , \end{aligned} \tag{4.17}$$

i.e. longitudinal component grows with energy. So any of the diagrams 1a,b is of the order of

$$Amp \sim (\frac{p_1}{m_W})(\frac{p_2}{m_Z})(\frac{p_3}{m_W})(\frac{p_4}{m_Z}) \sim \frac{s^2}{m_W^2 m_Z^2} \tag{4.18}$$

So in general theory of massive vector particle the scattering amplitude $W_L Z_L \to W_L Z_L$ grows quadratically with s. In the loops this dependence transforms into divergence of the loop integrals and this is why the theory of massive particle in general is nonrenormalizable.

But SM is a very special theory because it is gauge invariant. The sum of the diagram 1a and 1b (and the diagram 1c itself) is gauge invariant. It means that gauge boson interacts with conserved Noether currents. Consider for example the interaction of incoming W:

$$T_{1a} + T_{1b} = g_W \varepsilon_\mu^W(p_1) \ < Z, W | J_\mu^W | Z > \tag{4.19}$$

where $< Z, W|$, $< Z|$ represent the physical states of remaining particles. Since the J_μ^W is conserved current any matrix element of J_μ^W should be transversal

$$(p_1)_\mu < |J_\mu^W(p_1)| > \equiv 0 \qquad (4.20)$$

It means that in the case of the theory of massive particles with gauge-invariant interaction the sum of two diagrams 1a and 1b eq. (4.19) should be transversal with respect to W-boson momenta $(p_1)_\mu$. If so the most dangerous term $\sim p_\mu^1/m_W$ in polarization vector ε_μ^W gives zero contribution and the remaining terms $\sim m_W/\varepsilon$ gives

$$(T_{1a} + T_{1b}) \sim (\frac{m_W}{E}) < Z|J_\mu|ZW > \sim (\frac{m}{E})(\frac{E}{m})^3 \sim \frac{s}{m^2} \qquad (4.21)$$

The natural idea now is to kill in the same way the dangerous terms for polarization vector of two W-bosons and to prove that amplitude does not grow with energy. Unfortunately our trick works only once. The interaction of the W-bosons with corresponding conserved currents look like

$$T_{1a} + T_{1b} = g_W^2 \varepsilon_\mu^W(p_1) \varepsilon_\nu^W(p_3) \int d^4x_1 d^4x_2 e^{-ip_1x_1} e^{ip_3x_3} \times$$
$$\times < Z|T\{J_\mu^{W^+}(x_1) J_\nu^{W^-}(x_3)\}|Z > \qquad (4.22)$$

where $T\{\}$ is the T-product of two currents. Just because currents in non-abelian theory do not commute the T-product does not allow to cancel $p_{1\mu}$ and $p_{3\nu}$ – dangerous terms simultaneously.

Exercise 1. Prove this technical statement.

So we conclude that amplitudes without Higgs exchange grows linearly with s. Unitarity of the scattering amplitudes says that the any partial amplitude is less than 1. So sooner or later this amplitude has to violate unitarity bound. The value of energy $\sqrt{s_0}$ at which one of the partial waves intersects unitary bounds varies both with angular momenta l and from one process to another one. We are not going to repeat the detailed calculations made in literature. The result of these calculations gives

$$\sqrt{s_0} \sim 700 \text{ GeV} \qquad (4.23)$$

To stop the growth of the amplitude we have to switch on the diagrams with Higgs exchange. If we do it too late unitarity will be violated already. So to prevent the violation of unitarity bound

$$m_H \lesssim 700 \text{ GeV} \ . \qquad (4.24)$$

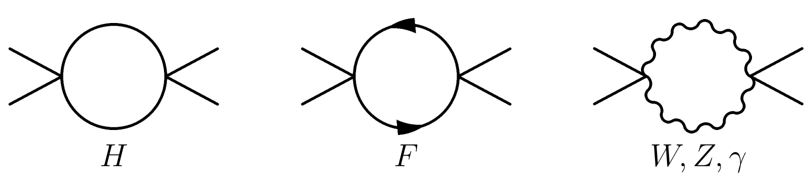
What is wrong with heavy Higgs? Actually the violation of unitarity that we found for tree diagrams means that the coupling constant becomes strong and that we can't restrict ourselves by calculation of tree diagrams, we have to include all loops and to calculate the whole infinite series in λ. We do not know how to deal with strong interaction. So the bounds (4.22), (4.23) and (4.24) say that for $m_H < 700$ GeV the theory can be treated perturbatively and for heavier Higgs we have a theory of strong interaction and we can't calculate $WZ \to WZ$ scattering theoretically. It is interesting to note that the attempts to work in strong coupling regime (e.g. using computer simulation for the Higgs theory on the lattice) demonstrate that the bound (4.23) and (4.24) is very stable, i.e. it takes place even for strong interacting Higgs.

4.3. Effective potential. Stability of the Universe and Bounds on m_H.

The Higgs potential in the SM

$$V_{\rm cl}(H) = \frac{\lambda}{4}H^4 - \frac{\mu^2}{2}H^2 \tag{4.25}$$

has minima that corresponds to nonzero v.e.v. of field H: $< H >_{\rm vac} = \eta$. Loop corrections change self-interactions of Higgs particles.



The effective potential that takes into account loops corrections was calculated by Coleman and Weinberg in 1973. In one-loop approximation it looks like

$$V_{\rm eff}(H) - V_{\rm cl}(H) = \frac{1}{64\pi^2}\left\{\frac{m_H^4 + 6m_W^4 + 3m_Z^4}{\eta^4} - \frac{12m_t^4}{\eta^4}\right\} H^4 \ln\frac{H^2}{M^2} \tag{4.26}$$

where we have neglected by small contributions from fermions other than t-quark. Note that due to Fermi-Dirac statistic the contribution of fermion loops has opposite sign in compared to the bosonic loops.

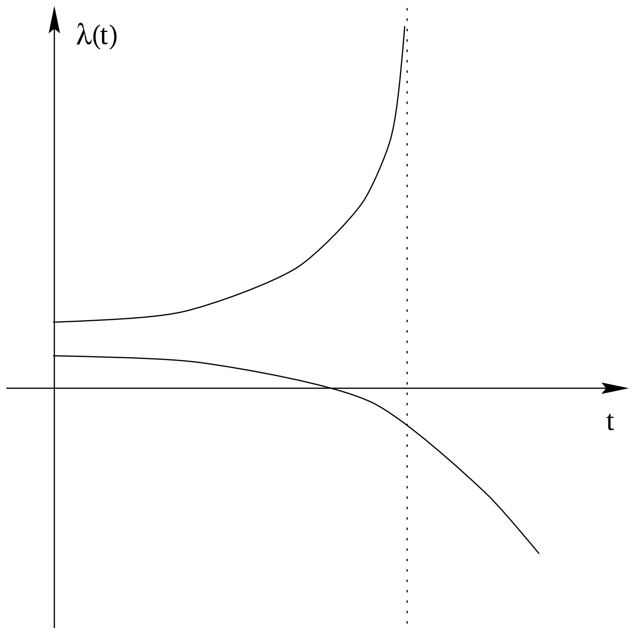


Figure 2:

It is clear that corrections (4.26) become more important than the main classical potential (4.25) for very large field H when $\left(\frac{m}{\eta}\right)^4 \ln H \gtrsim \left(\frac{m_H}{\eta}\right)^2$.

In one-loop approximation we get that the correction (4.26) has negative sign if $m_H < \sqrt[4]{12} m_t$. For this case the effective potential has no ground state(see fig. 2).

So even if our system was located first in the local minima at $< H >= \eta$ it will decay at $t \to \infty$ and the average value of field H will run to infinity. We know that nothing like that has happened with our Universe that is near 10^{10} years old. So the stability of the Universe imposes strong constraints on the masses of top and Higgs particles.

We should improve a little our one-loop formula (eq. (4.26)). For large H one-loop logarithmic corrections $\lambda \ln H$ and $\alpha_W \ln H$ are of the same order as tree terms, two-loop double-logarithmic terms are of the order of one-loop terms etc. So all these logarithmic terms should be taken into account. Fortunately this technical problem is not very difficult- renormalization group

techniques help to sum up such corrections. The result is

$$V^{\rm eff}(H) = -\frac{1}{2}\mu^2(t)H^2(t) + \frac{1}{4}\lambda(t)H^4(t) \qquad (4.27)$$

where $\lambda(t)$ and $\mu(t)$ are running parameters and $t = \ln H/\eta$. For small value of t (i.e. for small value of field H) the running parameters do not run far away from their classical values and the effective potential is equal to the classical one with the accuracy of small radiative corrections. For large H we can forget about μ^2H^2 and the whole dynamics at large H is governed by running coupling constant $\lambda(t)$. There are different contributions into differential equations for running $\lambda(t)$, coming from the loops with top quarks, vector bosons and Higgs boson itself. If the top quark contribution dominates, i.e. the higgs coupling to top (i.e. the top mass) is large, $\lambda(t)$ changes sign (see fig. 3) and the vacuum becomes unstable. This is reformulation of the phenomena that we had at one loop level.

If the Higgs selfinteraction dominates, i.e. the Higgs mass is large, then the evolution of $\lambda(t)$ is similar to the evolution of coupling in the H^4 theory without other fields. It is known that in this case the behavior of $\lambda(t)$ is

$$\lambda(t) = \frac{\lambda(t_0)}{1 - b\lambda(t_0)\ln\frac{t}{t_0}} \qquad (4.28)$$

and running coupling goes to infinity at some finite value of H (see fig. 3).

$$H = \Lambda \ .$$

This is the Landau pole in the running coupling constant. When initial condition $\lambda(t_0)$ (i.e. the value of Higgs mass) increases the value of Landau pole goes done. If we substitute this running coupling constant into eq.(4.27) we get that the effective potential runs to infinity at this value of H as well. This is some new phenomena.

Such singular behavior of the coupling constant is unacceptable from the physical point of view. Indeed for any <u>finite</u> value of the bare coupling constant λ^B (λ^B is equal to the running coupling $\lambda(t)$ at the cut-off Λ) we get that renormalized coupling constant (i.e. $\lambda(t)$ at low value of t) is equal to <u>zero</u>. It means that at low energy we get trivial free theory. This pathological theory seems to be unphysical.

There are two possibilities to improve bad behavior of $\lambda(t)$. The first one is so to say dynamical. For $\lambda \sim 1$ the multiloop corrections and nonper-

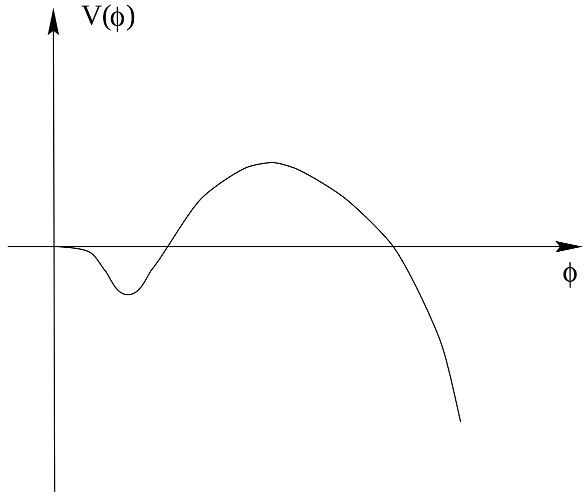


Figure 3:

turbative corrections change $\lambda(t)$ at large t so that Landau pole disappears. This is the possible solution of the problem in the strong coupling regime.

Another solution can be reached in perturbation theory if some new physics (i.e. new interactions and new particles) contribute into $\lambda(t)$ at scale near Λ so that pole disappears.

If we believe that there are no new physics up to some scale (or that the theory can be treated perturbatively up to this scale) we have to push the position of Landau pole Λ (calculated in one-loop approximation eq. (4.28)) to higher scale. This impose upper bound on the value of Higgs mass. So we have bounded m_H both from above and from below. This remarkable line of reasoning was invented by Cabibbo et al. in 1979.

There are different choices for the parameter Λ. For example Λ can be of the order of Planck scale

$$\Lambda \sim \Lambda_{Pl} = 10^{19} \text{ GeV} \quad ,$$

or of the order of Grand Unification Scale

$$\Lambda \sim \Lambda_{GUT} = 10^{15} \text{ GeV} \quad ,$$

or of the scale of the energy of the accelerator of the next generation

$$\Lambda \sim 10^{3} - 10^{5} \text{ GeV} \quad .$$

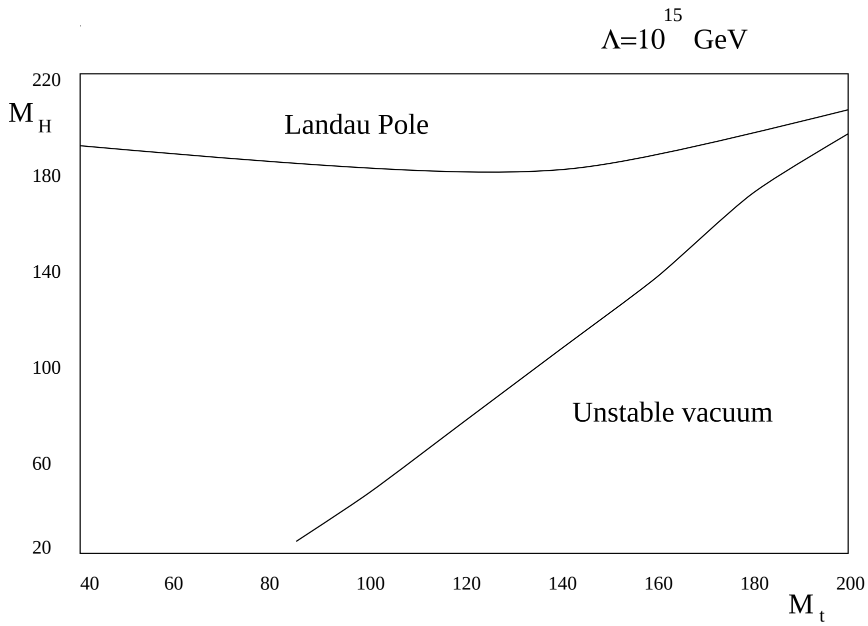


Figure 4:

We have to keep in mind all these possibilities. It is evident that for the strongest assumption that new physics does not appear up to the Planck scale we should get the strongest upper bound for m_H.

To derive more quantitative results we have to solve differential equation for the running coupling constant $\lambda(t)$. The renormalization of $\lambda(t)$ depends on self-interaction coupling, on gauge coupling and on Yukawa coupling constants. So we have to solve the whole system of coupled differential equations. This can be done numerically with the help of computer. The result of calculation for $\Lambda = 10^{15}$ GeV is presented in the fig. .

This is so to say the phase diagram in the plane m_t and m_H. Allowed region is located between two curves, the lower region corresponds to unstable vacuum and for the parameters in the upper region Landau pole appears at the scale lower than $\Lambda = 10^{15}$ GeV. For experimental value of $m_t \simeq 175$ GeV the allowed region for m_H is very strong

$$170 \text{ GeV} < m_H < 190 \text{ GeV} \quad (\Lambda = 10^{15} \text{ GeV}).$$

For $\Lambda \simeq 10^5$ GeV the upper bound is much weaker.

4.4. Basic properties of W and Z bosons.

The interactions of gauge bosons with quarks, leptons, Higgs bosons and with gauge bosons itself are determined by the principle of gauge invariance. In this section we are going to calculate the partial widths of W and Z boson.

According to eq. (3.21) we write the amplitude of the Z-boson decay into fermion-antifermion pair $\bar{f}f$ in the form

$$T(Z \to \bar{f}f) = \frac{e}{2sc} Z_\alpha \bar{\Psi}_f (g_V \gamma_\alpha + g_A \gamma_\alpha \gamma_5) \Psi_f \ , \tag{4.29}$$

where

$$g_V = T_3^f - 2s^2 Q^f$$
$$g_A = T_3^f$$
$$and$$
$$s = \sin\theta \ , \quad c = \cos\theta.$$

With good accuracy we can neglect the masses of fermions and derive the universal formula for any partial width of Z boson

$$\Gamma_Z \to \bar{f}f) = \Gamma_0 [g_V^2 + g_A^2] \ , \tag{4.30}$$

where Γ_0 is the standard width:

$$\Gamma_0 = \frac{G_F m_Z^3}{6\sqrt{2}\pi} = 332 \text{ MeV} \tag{4.31}$$

For neutrino we have

$$g_V = g_A = \frac{1}{2} \ ,$$

so that

$$\Gamma_{inv} = \sum_i \Gamma(Z \to \nu_i \tilde{\nu}_i) = 3 \cdot \Gamma_0 (\frac{1}{4} + \frac{1}{4}) = 498 \text{ MeV} \ . \tag{4.32}$$

This is theoretical prediction for the decay of Z boson into invisible modes.

For the decays to any of the pairs of charged leptons we have

$$g_A = -\frac{1}{2}$$

$$g_V = (-\frac{1}{2})(1 - 4s^2)$$

and

$$\Gamma_l = \Gamma(Z \to l\bar{l}) = \frac{1}{4}[1 + (1 - 4s^2)^2]\Gamma_0 \simeq 83.47 \text{ MeV} \tag{4.33}$$

Note that for $s^2 \simeq 0.2320$ the vector coupling g_V for charged lepton is a very small number!

For the decays into any of five pairs of quarks we have

$$\Gamma_q = \Gamma(Z \to \bar{q}q) = 3\Gamma_0[g_A^2 R_A + g_V^2 R_V] \tag{4.34}$$

where the factor 3 is due to three color of each quarks and the factors $R_{A,V}$ are due to exchange of gluons between quarks in the final state. In the first order of strong coupling $\hat{\alpha}_s = \alpha(q^2 = M_Z^2)$ they are equal

$$R_A = R_V = 1 + \frac{\hat{\alpha}_s}{\pi} \tag{4.35}$$

As a result

$$\Gamma_h = \Gamma(Z \to \text{hadrons}) = 3\Gamma_0 \left\{ 2[\frac{1}{4} + (\frac{1}{2} - \frac{4}{3}s^2)^2] + \right.$$

$$\left. +3[\frac{1}{4} + (\frac{1}{2} - \frac{2}{3}s^2)^2] \right\} (1 + \frac{\alpha_s}{\pi}) = 1676(1 + \frac{\hat{\alpha}_s}{\pi}) \text{ MeV} \tag{4.36}$$

The numerical value of $\hat{\alpha}_s$ found from deep-inelastic processes is of the order of $\hat{\alpha}_s \simeq 0.119$. So the corrections due to strong interaction in the final state are of the order of 4%.

For the total width we have

$$\Gamma_Z^{th} = \Gamma_{inv} + 3\Gamma_l + \Gamma_h \simeq 2488.5\text{MeV} \tag{4.37}$$

that should be compared with the experimental LEP and SLC value

$$\Gamma_Z^{ex} = 2493.9 \pm 2.4 \text{ MeV} \ \ . \tag{4.38}$$

Though the agreement between Γ_Z^{th}, calculated in the tree approximation, and Γ_Z^{ex} is very good the experimental accuracy is better. So we have to improve the accuracy of the theoretical calculation and to take into account the radiative corrections (see lecture V).

Consider now the decays of W boson. According to eq. (3.23) the amplitude of W^+ decay into $\tilde{\nu}_l$l can be written in the form

$$T(W \to \tilde{\nu}_l l) = \frac{e}{2\sqrt{2}s} W_\alpha(\bar{l}\gamma_\alpha(1 + \gamma_5)\nu) \tag{4.39}$$

Neglecting leptonic mass we get

$$\Gamma(W \to l\tilde{\nu}) = \frac{1}{6\sqrt{2}\pi} G_\mu m_W^3 \simeq 226 \text{ MeV} \ . \tag{4.40}$$

Consider now the hadronic decays of W-boson. The decay width into top quark is equal to zero since top quark is heavier than W-boson. With good accuracy we can neglect the masses of other quark. In this approximation

$$\Gamma(W \to \text{hadrons}) = 3 \cdot 2\Gamma(W \to e\tilde{\nu}) \tag{4.41}$$

where factor 3 is due to color of each quark and factor 2 is due to two quark channels of W decay.

Exercise: Prove that in massless approximation hadronic width eq. (4.41) does not depend on the CKM angles.

In this approximation the theoretical prediction for the total width is

$$\Gamma_W^{th} \simeq 9\Gamma(W \to \tilde{\nu}_l l) \simeq 2.03 \text{ GeV} \tag{4.42}$$

that has to be compared with the experimental value

$$\Gamma_W^{exp} \simeq 2.06 \pm 0.06 \text{ GeV} \ . \tag{4.43}$$

As it was expected the agreement is of the order of percent. The experimental accuracy is of the same order.

Lecture V. **Radiative Correction in SM.**
Hunting for virtual t-quark.

5.0. Z-physics at LEP and SLC.

To test the predictions of the SM the huge "factories" of Z-bosons (e^+e^- colliders) were constructed of CERN (LEPI) and at SLAC (SLC). Electrons and positrons in thus colliders collide at the centre of mass energy equal to the Z-boson mass. The reactions that are studied can be presented in the form

$$e^+e^- \to Z \to \bar{f}f$$

where

$$\bar{f}f = \begin{cases} \nu\tilde{\nu} & \quad\quad \textit{invisible modes} \\ l\bar{l} & \quad\quad \textit{charged leptons} \\ q\bar{q} & \quad\quad \textit{hadrons.} \end{cases}$$

The LEPI was terminated in the fall 1995 in order to give place to LEP II. Four groups — collaborations ALEPH, DELPHI, L3, OPAL — have collected near $2 \cdot 10^7$ Z-bosons. The SLC has worked from the fall 1989 till the fall 1998. The SLD detector recorded near $5 \cdot 10^5$ Z-bosons. The electrons in the SLC were longitudinally polarized so that SLD group could provide very high precision data having relatively low statistics. More than two thousand experimentalists and engineers and hundreds of theorists participated in this unique project – one of the largest project in the history of physics.

Near the dozen of independent observables were measured with fantastic precision of the order of 10^{-3} (10^{-5} for the case of Z-boson mass). The scale of the radiative corrections in the SM is of the order of $\alpha_{W,Z}/\pi \sim 10^{-2} - 10^{-3}$. Therefore LEP-I and SLD data provide precision test of the SM as a renormalizable field theory, i.e. with loops included.

The theoretical study of electroweak corrections in SM started in 1970's and was elaborated by a number of theoretical groups. Near 10^2 theorists had published near $2 \cdot 10^3$ papers on radiative corrections (see e.g.ref.[2]). This study has been summarized in Yellow CERN Reports of Working Groups on precision calculations for the Z resonance in 1995 [3]. The deviations in theoretical calculations of different groups are by the order of magnitude smaller than the experimental uncertainties.

By comparing the $\Gamma_{invisible}$ with theoretical predictions for neutrino decays the result of fundamental importance was established – the sequence of the generations with light neutrino is completed with number of generation $N_f = 3$.

The best fit of the most recent data submitted to Vancouver (1998) conference is presented at Table I.

This fit gives

$$m_t = 171.6 \pm 4.9\ Gev$$

$$m_H = 159.1^{+144}_{-83}\ Gev$$

$$\hat{\alpha}_s(m_z) = 0.119 \pm 0.003$$

$$\chi^2/n.d.f. = 16.4/15$$

The quality of this fit is very good. We conclude that the SM gives the perfect description of Z physics. New physics can not improve the fit of LEP and SLC data. Thus the Standard Model has been confirmed up to the loop corrections. What is more important is that the loop corrections can be used to gather data on the not yet discovered particle. For instance, even before t-quark was discovered at Tevatron, its mass was predicted by analyzing the loops and LEP-SLC data. The hunting for virtual top quark is a very bright example of the collaboration of the theory with the experiment.

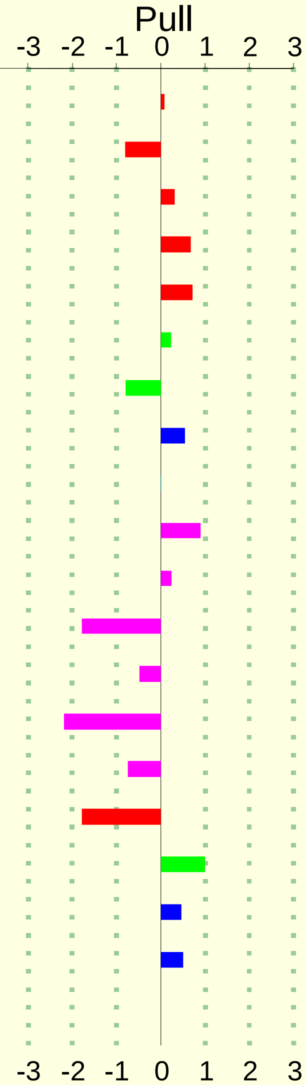
5.1. Decoupling of heavy flavors from Low-Energy Physics in QED and QCD.

It is interesting to understand why in 1950's nobody worried about the contribution of top quark (and other heavy flavours) into magnetic moment of the electron known with very high accuracy. The answer to this question is that for $q \sim m_e$ the corrections due to top quark are suppressed as a power of (m_e^2/m_t^2) i.e. the contribution was negligible. In QED we have decoupling of heavy (unknown) particles from the low-energy observables. Why? Consider the contribution of t-quark into QED observables. The only diagram with t-quarks in loop is the self-energy of the photon

$$\Pi_{\mu\nu}(q) = i\langle 0|\{j_\mu(q), j_\nu(-q)\}|0\rangle \tag{5.1}$$

Pulls from SM fit to all data

	Measurement	Pull
m_Z (GeV)	91.1867 ± 0.0021	.08
Γ_Z (GeV)	2.4939 ± 0.0024	-.80
σ^0_{hadr} (nb)	41.491 ± 0.058	.31
R_e	20.765 ± 0.026	.66
$A_{fb}^{0,e}$	0.01683 ± 0.00096	.72
A_e	0.1479 ± 0.0051	.24
A_τ	0.1431 ± 0.0045	-.80
$\sin^2\theta_{eff}^{LEPT}$	0.2321 ± 0.0010	.54
m_W (GeV)	80.370 ± 0.090	.01
R_b	0.21656 ± 0.00074	.90
R_c	0.1733 ± 0.0044	.24
$A_{fb}^{0,b}$	0.0991 ± 0.0021	-1.78
$A_{fb}^{0,c}$	0.0714 ± 0.0044	-.47
A_b	0.856 ± 0.036	-2.18
A_c	0.638 ± 0.040	-.74
$\sin^2\theta_{eff}^{LEPT}$	0.23101 ± 0.00031	-1.78
$\sin^2\theta_W$	0.2255 ± 0.0021	1.06
m_W (GeV)	80.410 ± 0.090	.45
m_t (GeV)	173.8 ± 5.0	.50
$1/\alpha$	128.896 ± 0.090	-.04



$$\chi^2 / dof = 16.4 / 15$$

where $j_\mu(q)$ is the electromagnetic current of t-quark. Self energy has dimension 2: $[\Pi_{\mu\nu}] = m^2$. So one can expect that there exit terms of the order of

$$\Pi_{\mu\nu} \sim \alpha m_t^2 g_{\mu\nu}$$

This expectation is wrong in the case of <u>conserved</u> currents

$$q_\mu j_\mu(q) = 0 \tag{5.2}$$

Indeed for conserved current the self-energy operator should be transversal $q_\mu \Pi_{\mu\nu} = 0$. So

$$\Pi_{\mu\nu}(q) = (q_{\mu\nu} q^2 - q_\mu q_\nu)\Pi(q^2), \tag{5.3}$$

Equation (5.3) implies that the photon remains massless. The scalar function $\Pi(q^2)$ has dimension zero and the only possible contribution of t-quark into $\Pi(q^2)$ can be written in the form

$$\Pi(q^2) \sim \alpha \ln \frac{\Lambda^2}{m_t^2 + q^2}$$

where Λ is cut-off. The self-energy keeps the memory of heavy flavours!

The crucial step is renormalization. For instance consider the Coulomb scattering. If we take into account the infinite chain of self-energy contribution into the photon propagator we get for Coulomb amplitude

$$T_{Coulomb} = \frac{e_0^2(\Lambda)}{q^2(1 + \Pi(q^2))} \tag{5.4}$$

At low q^2 we reproduce the Coulomb-low

$$T = \frac{e_{phys}^2}{q^2} \tag{5.5}$$

with

$$e_{phys}^2 = \frac{e_0^2(\Lambda)}{1 + \Pi(0)} \tag{5.6}$$

When we rewrite the amplitude (5.4) in terms of e_{phys}^2 we get

$$T \simeq \frac{e^2_{phys}}{q^2[1+\Pi(q^2)-\Pi(0)]} \qquad (5.7).$$

As a result:
1) the dependence on cut-off Λ disappears

$$\Delta\Pi = \Pi(q^2) - \Pi(0) \sim \alpha \ln \frac{m_t^2}{m_t^2 + Q^2};$$

2) the contribution of heavy flavour is suppressed as a power (q^2/m_t^2):

$$\Delta\Pi \sim -\alpha \left(\frac{q^2}{m_t^2}\right) \to 0.$$

This is the sample of the famous decoupling theorem. It works for the theories with conserved vector currents.

5.2. Non-decoupling of chiral matter. Heavy Flavor contribution into electroweak observables.

In the Standard Model the left components of t- and b-quarks belong to $SU(2)_W$ doublet representation: $Q_L = \binom{t_L}{b_L}$. Therefore for the case when $m_t \gg m_b$ and for small energies $E \leq m_t$ we have effectively the explicit violation of $SU(2)_W$ symmetry. For the virtual momenta $q \sim \Lambda \sim m_t$ theory looks like the old nonrenormalizable theory. It mean that one-loop corrections diverge quadratically $\delta_1 \sim \alpha\Lambda^2/m_Z^2$, two-loop corrections diverge quartically $\delta_2 \sim (\alpha\Lambda^2/m_Z^2)^2$. So we expect that the corrections to the low-energy observables due to top contribution are of the order of

$$\delta_1 \sim \alpha_W t$$

$$\delta_2 \sim \alpha_W^2 t^2$$

where $t = m_t^2/m_Z^2$, i.e.corrections are not suppressed, they grow with top mass m_t. Heavy flavours are not decoupled from the low-energy observables for the chiral matter. As a result the radiative corrections in the SM are sensitive to the top contribution.

Consider for example the ratio of g_V and g_A for the decay $Z \to \bar{l}l$. It is rather simple exercise to calculate this ratio

$$R_l = \left(\frac{g_V}{g_A}\right)_l = 1 - 4s^2 - \frac{3\bar{\alpha}}{4\pi(c^2 - s^2)}(t + \delta V_R) \tag{5.8}$$

The linear term $\sim t$ originates from different self-energies with top quark in the loops and δV_R is the contribution of the rest of the diagrams (a few dozens ones) that do not depend on t.

If we compare experimental value of t

$$t^{\rm exp} \simeq 3.7$$

with experimental value of $(t + \delta V_R)$

$$(t + \delta V_R)^{\rm exp} \simeq -0.49 \pm 0.32$$

we find that the constant terms are as much important as a linear term.

So we have to perform the accurate calculation of the whole set of one-loop diagrams.

5.3. Radiative corrections in SM.

a) Strategy.

There are 3 steps in the calculations of the radiative corrections.

1) Calculate all observables in terms of base parameters g_1^B, g_2^B, η^B and cut-off Λ.

2) Find 3 basic observables and express base parameters in terms of these 3 physical observables and cut-off Λ.

3) Substitute these expressions into formulas for other observables. Dependence on cut-off disappears.

b) Basic parameters.

It is reasonable to calculate observables in terms of quantities that are known to the highest degree of precision.

$$\alpha^{-1} = 137.035985(61)$$

$$G_\mu = 1.16639(2) \cdot 10^{-5} Gev^{-2} \tag{5.9}$$

$$m_Z = 91.1867(21) Gev$$

c) The Choice of Born approximation

In contrast to $\alpha(q^2)$ that is running coupling constant, the two electroweak coupling constants $\alpha_Z(q^2)$ and $\alpha_W(q^2)$ are not "running" but "crawling" for $q^2 \leq m_Z^2$.

The natural scale for electroweak physics is $q^2 = m_Z^2$. Therefore it is evident that $\bar{\alpha} \equiv \alpha(m_Z^2)$, not $\alpha \equiv \alpha(0)$ is relevant parameter in electroweak physics. The value of $\bar{\alpha}$ is less accurate.

$$\bar{\alpha} = \alpha(m_Z^2) = [128.896 \pm 0.090]^{-1} \tag{5.10}$$

With this parameterization it is convenient to introduce the weak angle θ by the relation

$$G_\mu = \frac{\pi\bar{\alpha}}{\sqrt{2}m_Z^2 s^2 c^2} \tag{5.11}$$

where $s^2 = \sin^2\theta$, $c^2 = \cos^2\theta$. Numerically

$$s^2 = 0.2311(22) \tag{5.12}$$

d) Basic relations for electroweak observables.

A simple calculation gives the following result for one-loop electroweak corrections in the case gluon free observables:

$$\frac{m_W}{m_Z} = c\left[1 + \frac{3\bar{\alpha}}{32\pi s^2(c^2 - s^2)}(t + \delta V_m)\right]$$
$$g_{Al} = -\frac{1}{2} - \frac{3\bar{\alpha}}{64\pi s^2 c^2}(t + \delta V_A) \tag{5.13}$$
$$R_l = (g_V/g_A)_l = 1 - 4s^2 + \frac{3\bar{\alpha}}{4\pi(c^2 - s^2)}(t + \delta V_R)$$

There are a few diagrams that contribute into leading term $\sim t$ and there are dozens of them for nonleading δV.

The hadronic decays of Z are more sensitive to the value of the gluonic coupling constant α_s.

$$\Gamma(Z \to q\bar{q}) = \frac{G_\mu m_Z^3}{2\sqrt{2}\pi}[g_A^2 R_A + g_V^2 R_V] \tag{5.14}$$

The "radiator" $R_{A,V}$ contain QCD and QED corrections caused by the final state emission and exchange of gluons and photons. The are hundreds diagrams that contribute into $R_{A,V}$.

5.4. Hunting for virtual top. Great success.

A comparison of the experimental data with the result of theoretical calculation eq.(5.13) led to the prediction of the t-quark mass m_t.

$$m_t = 180(5)^{+17}_{-20} \quad Gev$$

where the number in parentheses is the uncertainty due the uncertainties of the data. The center value corresponds to the assumption that $m_H = 300$ Gev, the upper and lower "shifts" correspond to $m_H = 1000$ Gev and 60 Gev, respectively.

The best fit of all observables gives

$$(m_t) \simeq 173.8 \pm 5.3 \quad Gev$$

These numbers are in perfect agreement with the recent direct measurement of the top-quark mass by two collaboration at FNAL

$$m_t = 173.8 \pm 5.0 \quad Gev.$$

5.5. Hopeless hunting for virtual Higgs.

Consider the limit of very large Higgs boson mass m_H. For $E \ll m_H$ we have $SU(2)$ symmetric theory of massive gauge bosons, i.e. effectively nonrenormalizable theory. As I explained in the Lecture II due to the gauge symmetry the leading divergence of the loop disappears. So the one-loop corrections diverge logarithmically

$$\delta_1 \sim \alpha_W \ln \frac{\Lambda^2}{m_Z^2} \sim \alpha_W \ln \frac{m_H^2}{m_Z^2} = \alpha_W \ln h$$

two-loop corrections diverge quadratically

$$\delta_2 \sim \alpha_W^2 \left(\frac{\Lambda^2}{m_Z^2}\right) \sim \alpha_W^2 h.$$

Here $h = m_H^2/m_Z^2$.

This is the famous Veltman screening theorem. The weak dependence of radiative corrections on h results in a rather poor accuracy for m_H derived from the precision data. The central value of m_H from the fit should not be taken seriously. It is very unstable. Any tiny corrections or any change of the parameter can shift it by the order of magnitude.

The one sigma upper bound is more reliable. According to the recent fit

$$m_H < 280 Gev \qquad\qquad 95\% \; c.l.$$

It seems that the fit of the precision data is not the best way for hunting for Higgs boson.

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